Published by Institute of Physics Publishing for SISSA

Received: *March 6, 2006*
Accepted: *April 10, 2006*
Published: *April 20, 2006*

Open string modes at brane intersections

Daniel Areán and Alfonso V. Ramallo

Departamento de Física de Partículas, Universidade de Santiago de Compostela and Instituto Galego de Física de Altas Enerxías (IGFAE)
E-15782 Santiago de Compostela, Spain
E-mail: arean@fpaxp1.usc.es, alfonso@fpaxp1.usc.es

Abstract: We study systematically the open string modes of a general class of BPS intersections of branes. We work in the approximation in which one of the branes is considered as a probe embedded in the near-horizon geometry generated by the other type of branes. We mostly concentrate on the D3-D5 and D3-D3 intersections, which are dual to defect theories with a massive hypermultiplet confined to the defect. In these cases we are able to obtain analytical expressions for the fluctuation modes of the probe and to compute the corresponding mass spectra of the dual operators in closed form. Other BPS intersections are also studied and their fluctuation modes and spectra are found numerically.

Keywords: AdS-CFT Correspondence, Gauge-gravity correspondence, D-branes, Brane Dynamics in Gauge Theories.

JHEP04(2006)037

© SISSA 2006

Contents

JHEP04(2006)037

---

1. Introduction

The origin of the gauge/gravity correspondence is the twofold description of D-branes [1, 2]. On one hand the D-branes have an open string description as hypersurfaces on which open strings can end. The dynamics of these hypersurfaces can be described by the (super) Yang-Mills theory in flat space. On the other hand, the D-branes also appear as solitons of the type II low energy closed string effective action and are solutions of the classical equations of supergravity. By relating these two descriptions one can get information of the quantum dynamics of gauge theories by studying classical supergravity.

In its original form, all matter fields of the gauge theory side of the correspondence are in the adjoint representation. Clearly, if we want to apply this duality to more realistic scenarios we should be able to obtain a holographic description of theories with matter in the fundamental representation i.e. quarks. This can be achieved by adding open strings to the supergravity side of the correspondence. The simplest way to do this is by considering fundamental strings whose ends are fixed at the UV, as was done in ref. [3] to compute the expectation values of the Wilson loop operators. Notice that the fundamentals introduced in this way are external static quark sources.

Alternatively, one can try to generalize the gauge/gravity correspondence by adding brane probes embedded in supergravity backgrounds. The fluctuations of the probe correspond to degrees of freedom of open strings connecting the brane probe and those that generated the background [4]. On the field theory side these open strings are identified with fundamental hypermultiplets of dynamical quarks whose masses are proportional to the distance between the two types of branes.

JHEP04(2006)037

One can use this setup to add flavor to some supergravity duals [5]. In particular, for the $AdS_5 \times S^5$ geometry the appropriate flavor branes are D7-branes which fill the spacetime directions of the gauge theory and are extended along the holographic direction [6]. The starting point in this construction are two stacks of D3- and D7-branes which intersect along three common spatial directions. If the number of D3-branes is large we can take the decoupling limit and substitute them by the $AdS_5 \times S^5$ geometry. Moreover, when the number of D7-branes is small compared to the number of D3-branes, we can assume that the D7-branes do not backreact on the geometry and treat them as a probe whose fluctuation modes are identified with the mesons of the dual gauge theory. Remarkably, the mass spectrum of the complete set of fluctuations can be obtained analytically and the identification between the different modes and the dual operators can be carried out [7]. Different flavor branes and their spectra for several backgrounds have been considered in the recent literature (see [8]–[20]).

In this paper we generalize these results for the D3-D7 system to a general class of BPS intersections of two types of branes, both in type II theories and in M-theory. In our approach the lower dimensional brane is substituted by the corresponding near-horizon geometry, while the higher dimensional one will be treated as a probe. Generically, the addition of the probes to the supergravity background creates a defect in the gauge theory dual in which extra hypermultiplets are localized. In the decoupling limit one sends the string scale l_s to zero keeping the gauge coupling of the lower dimensional brane fixed. It is straightforward to see that the gauge coupling of the higher dimensional brane vanishes in this limit and, as a consequence, the corresponding gauge theory decouples and the gauge group of the higher dimensional brane becomes the flavor symmetry of the effective theory at the intersection.

The prototypical example of a defect theory is the one dual to the D3-D5 intersection. This system was proposed in ref. [4] as a generalization of the usual AdS/CFT correspondence in the $AdS_5 \times S^5$ geometry. Indeed, if the D5-branes are at zero distance of the D3-branes they wrap an $AdS_4 \times S^2$ submanifold of the $AdS_5 \times S^5$ background. It was argued in ref. [4] that the AdS/CFT correspondence acts twice in this system and, apart from the holographic description of the four dimensional field theory on the boundary of AdS_5, the fluctuations of the D5-brane probe should be dual to the physics confined to the boundary of AdS_4.

The field theory dual of the D3-D5 intersection corresponds to $\mathcal{N} = 4$, $d = 4$ super Yang-Mills theory coupled to $\mathcal{N} = 4$, $d = 3$ fundamental hypermultiplets localized at the defect. In ref. [21] the action of this model in the conformal limit of zero D3-D5 separation was constructed and a precise dictionary between operators of the field theory and fluctuation modes of the probe was obtained (see also refs. [22, 23]). We will extend these results to the case in which the distance between the D3- and D5-branes is non-zero. This non-vanishing distance breaks conformal invariance by giving mass to the fundamental hypermultiplets. Interestingly, the differential equations for the quadratic fluctuations can be decoupled, solved analytically and the corresponding mass spectra can be given in closed form. These masses satisfy the degeneracy conditions expected from the structure of the supersymmetric multiplets found in ref. [21].

JHEP04(2006)037

The D3-D5 intersection can be generalized to the case of a Dp-D(p+2) BPS intersection, in which the D(p+2)-brane creates a codimension one defect in the (p+1)-dimensional gauge theory of the Dp-brane. The differential equations of the fluctuations can also be decoupled in this more general case. Even if we will not be able to solve these equations in analytic form for $p \neq 3$, we will disentangle the mode structure and we shall find the corresponding mass spectra by numerical methods. We will see that the numerical values of the masses satisfy degeneracy relations which are very similar to the ones found for the exactly solvable D3-D5 system.

Another interesting case of defect theory arises from the D3-D3 BPS intersection, in which the two D3-branes share one spatial dimension. In the conformal limit this intersection gives rise to a two-dimensional defect in a four-dimensional CFT. In this case one has, in the probe approximation, a D3-brane probe wrapping an $AdS_3 \times S^1$ submanifold of the $AdS_5 \times S^5$ background. In ref. [24] the spectrum of fluctuations of the D3-brane probe in the conformal limit was obtained and the corresponding dual fields were identified (see also [25]). Notice that in this intersection both types of branes have the same dimensionality and the decoupling argument explained above does not hold anymore. Therefore, it is more natural to regard this system as describing two $\mathcal{N} = 4$ four-dimensional theories coupled to each other through a bifundamental hypermultiplet living on the two-dimensional defect. This fact is reflected in the appearance of a Higgs branch in the system, in which the two types of D3-branes merge along some holomorphic curve. We will study this system when a non-zero mass is given to the hypermultiplet. Again, we will be able to solve analytically the differential equations for the fluctuations and to get the exact mass spectrum of the model. This system generalizes to the case of a Dp-Dp intersection, in which the two Dp-branes have $p-2$ common spatial directions. For $p \neq 3$ we will get the mass spectrum of the different modes from a numerical integration of the differential equations of the fluctuations.

The D3-D7 intersection described above corresponds to a codimension zero "defect". This configuration is a particular case of the Dp-D(p+4) BPS intersection in which the D(p+4)-brane fills completely the (p+1)-dimensional worldvolume of the Dp-brane and acts as a flavor brane of the corresponding supersymmetric gauge theory in p+1 dimensions. Again, for $p \neq 3$ one has to employ numerical methods to get the mass spectrum.

This paper is organized as follows. In section 2 we will consider a general intersection of two branes of arbitrary dimensionalities. By placing these two branes at a non-zero distance, and by imposing a no-force condition on the static configuration, we get an equation which determines the BPS intersections. Next, we consider fluctuations of the scalars transverse to both types of branes around the static BPS configurations. For D-brane probes embedded in $AdS_5 \times S^5$ the corresponding differential equation can be reduced, after a change of variables, to the hypergeometric differential equation. Thus, in these cases the form of the fluctuations can be obtained analytically and the mass spectra of the transverse scalar fluctuations can be found by imposing suitable boundary conditions at the UV. In the general case the fluctuation equation can be transformed into the Schrödinger equation for some potential, which allows to apply the WKB method to get an estimate of the mass levels.

JHEP04(2006)037

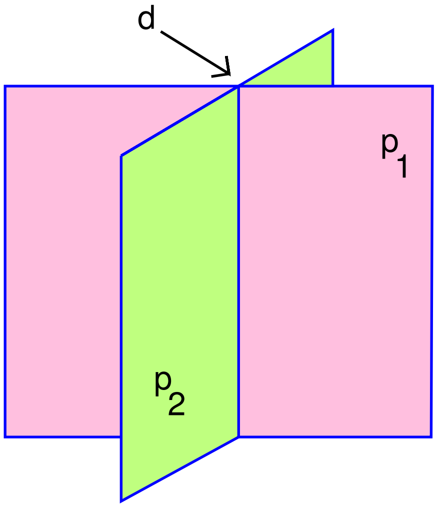

Figure 1: A general orthogonal intersection of a p_1- and p_2-brane along d spatial directions.

In section 3 we study in detail the complete set of fluctuations, involving all scalar and vector worldvolume fields, of the D3-D5 intersection. In general, these fluctuations are coupled to each other and one has to decouple them in order to get a system of independent equations. The decoupling procedure is actually the same as in the more general Dp-D(p+2) intersection and is given in detail in appendix B. In section 3 we use this procedure to get the exact mass spectrum of the D3-D5 system. We also recall the fluctuation/operator dictionary found in ref. [21] and we check that the masses we find for the modes are consistent with the arrangement of the dual operators in supersymmetric multiplets.

In section 4 we perform a complete analysis of the exactly solvable D3-D3 intersection. In this case the differential equations can also be decoupled and solved in terms of the hypergeometric function. As a consequence, the exact mass spectrum can be found and matched with the fluctuation/operator dictionary established in ref. [24]. We will also show the appearance of the Higgs branch and how it is modified by the fact that the hypermultiplet is massive.

In the main text we will concentrate on the study of the exactly solvable intersections and we have left other cases to the appendices. These cases include the Dp-D(p+2), Dp-Dp, Dp-D(p+4) and F1-Dp intersections of the type II theory, as well as the M2-M2, M2-M5 and M5-M5 intersections of M-theory. In all of them we compute the numerical mass spectra and their WKB estimates. Finally, in section 5 we summarize our results and point out some possible extensions of our work.

2. Fluctuations of intersecting branes

Let us consider an orthogonal intersection of a p_1-brane and a p_2-brane along d common spatial directions ($p_2 \geq p_1$), as depicted in figure 1. We shall denote this intersection, both in type II string theory and M-theory, as $(d|p_1 \perp p_2)$. We shall treat the lower

JHEP04(2006)037

dimensional p_1-brane as a background, whereas the p_2-brane will be considered as a probe. The background metric will be taken as:

$$ds^2 = \left[\frac{r^2}{R^2}\right]^{\gamma_1} (-dt^2 + (dx^1)^2 + \cdots + (dx^{p_1})^2) + \left[\frac{R^2}{r^2}\right]^{\gamma_2} d\vec{y}\cdot d\vec{y} \,, \tag{2.1}$$

where R, γ_1 and γ_2 are constants that depend on the case considered, $\vec{y} = (y^1, \ldots, y^{D-1-p_1})$ with D=10, 11 and $r^2 = \vec{y}\cdot\vec{y}$. In the type II theory the supergravity solution also contains a dilaton ϕ, which we will parametrize as:

$$e^{-\phi(r)} = \left[\frac{R^2}{r^2}\right]^{\gamma_3} \,, \tag{2.2}$$

with γ_3 being constant (in the case of a background of eleven dimensional supergravity we just take $\gamma_3 = 0$).

Let us now place a p_2-brane in this background extended along the directions:

$$(t, x^1, \ldots, x^d, y^1, \ldots, y^{p_2-d}) \,. \tag{2.3}$$

We shall denote by $\vec{z}$ the set of y coordinates transverse to the probe:

$$\vec{z} = (z^1, \ldots, z^{D-p_1-p_2+d-1}) \,, \tag{2.4}$$

with $z^m = y^{p_2-d+m}$ for $m = 1, \ldots, D - p_1 - p_2 + d - 1$. Notice that the $\vec{z}$ coordinates are transverse to both background and probe branes. Moreover, we shall choose spherical coordinates on the p_2-brane worldvolume which is transverse to the p_1-brane. If we define:

$$\rho^2 = (y^1)^2 + \cdots + (y^{p_2-d})^2 \,, \tag{2.5}$$

clearly, one has:

$$(dy^1)^2 + \cdots (dy^{p_2-d})^2 = d\rho^2 + \rho^2 d\Omega^2_{p_2-d-1} \,, \tag{2.6}$$

where $d\Omega^2_{p_2-d-1}$ is the line element of a unit (p_2-d-1)-sphere. Obviously we are assuming that $p_2 - d \geq 2$.

2.1 BPS intersections

Let us consider first a configuration in which the probe is located at a constant value of $|\vec{z}|$, i.e. at $|\vec{z}| = L$ (see figure 2). If ξ^a are a set of worldvolume coordinates, the induced metric on the probe worldvolume for such a static configuration will be denoted by:

$$ds^2_I = \mathcal{G}_{ab}\, d\xi^a d\xi^b \,. \tag{2.7}$$

In what follows we will use as worldvolume coordinates the cartesian ones $x^0 \cdots x^d$ and the radial and angular variables introduced in eqs. (2.5) and (2.6). Taking into account that, for an embedding with $|\vec{z}| = L$, one has $r^2 = \rho^2 + \vec{z}^{\,2} = \rho^2 + L^2$, the induced metric can be written as:

$$ds^2_I = \left[\frac{\rho^2 + L^2}{R^2}\right]^{\gamma_1} (-dt^2 + (dx^1)^2 + \cdots + (dx^d)^2) + \left[\frac{R^2}{\rho^2 + L^2}\right]^{\gamma_2} (d\rho^2 + \rho^2 d\Omega^2_{p_2-d-1}) \,. \tag{2.8}$$

JHEP04(2006)037

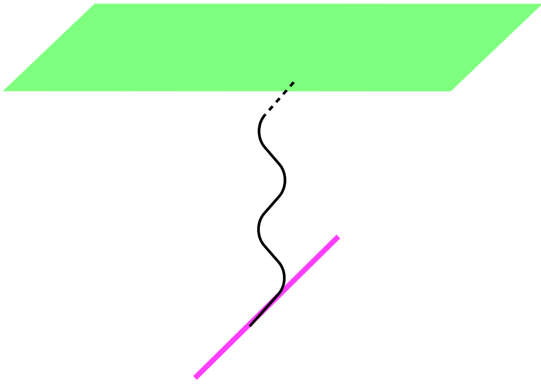

Figure 2: The two branes of the intersection are separated a finite distance. In the figure one of the branes is represented as a one-dimensional object. An open string can be stretched between the two branes.

The action of the probe is given by the Dirac-Born-Infeld action. In the configurations we study in this section the worldvolume gauge field vanishes and it is easy to verify that the lagrangian density reduces to:

$$\mathcal{L} = -e^{-\phi}\sqrt{-\det \mathcal{G}}\ . \tag{2.9}$$

For a static configuration such as the one with $|\vec{z}| = L$, the energy density $\mathcal{H}$ is just $\mathcal{H} = -\mathcal{L}$. By using the explicit form of $\mathcal{G}$ in (2.8), one can verify that, for the $|\vec{z}| = L$ embedding, $\mathcal{H}$ is given by:

$$\mathcal{H} = \left[\frac{\rho^2 + L^2}{R^2}\right]^{\frac{\gamma_1}{2}(d+1) - \frac{\gamma_2}{2}(p_2-d) - \gamma_3} \rho^{p_2-d-1}\sqrt{\det \tilde{g}}\ , \tag{2.10}$$

where $\tilde{g}$ is the metric of the unit $(p_2 - d - 1)$-sphere. In a BPS configuration the no-force condition of a supersymmetric intersection requires that $\mathcal{H}$ be independent of the distance L between the branes. Clearly, this can be achieved if the γ_i-coefficients are related as:

$$\gamma_3 = \frac{\gamma_1}{2}(d+1) - \frac{\gamma_2}{2}(p_2 - d)\ . \tag{2.11}$$

Let us rewrite this last equation as:

$$d = \frac{\gamma_2}{\gamma_1 + \gamma_2}\, p_2 + \frac{2\gamma_3 - \gamma_1}{\gamma_1 + \gamma_2}\ , \tag{2.12}$$

which gives the number d of common dimensions of the intersection in terms of the parameters γ_i of the background and of the dimension p_2 of the probe brane. In the following subsections we shall consider some particular examples.

2.1.1 Dp-brane background

In the string frame, the supergravity solution corresponding to a Dp-brane with $p < 7$ has the form displayed in eqs. (2.1) and (2.2) with $p_1 = p$, R given by

$$R^{7-p} = 2^{5-p}\,\pi^{\frac{5-p}{2}}\,\Gamma\Big(\frac{7-p}{2}\Big)\, g_s\, N\, (\alpha')^{\frac{7-p}{2}}\ , \tag{2.13}$$

JHEP04(2006)037

and with the following values for the exponents γ_i:

$$\gamma_1 = \gamma_2 = \frac{7-p}{4} \ , \qquad \qquad \gamma_3 = \frac{(7-p)(p-3)}{8} \ . \tag{2.14}$$

Moreover, the Dp-brane solution is endowed with a Ramond-Ramond $(p{+}1)$-form potential, whose component along the Minkowski coordinates $x^0 \cdots x^p$ can be taken as:

$$\left[C^{(p+1)}\right]_{x^0\cdots x^p} = \left[\frac{r^2}{R^2}\right]^{\frac{7-p}{2}} \ . \tag{2.15}$$

Applying eq. (2.12) to this background, we get the following relation between d and p_2:

$$d = \frac{p_2 + p - 4}{2} \ . \tag{2.16}$$

Let us now consider the case in which the probe brane is another D-brane. As the brane of the background and the probe should live in the same type II theory, $p_2 - p$ should be even. Since $d \le p$, we are left with the following three possibilities:

$$(p|Dp \perp D(p+4)) \ , \qquad (p-1|Dp \perp D(p+2)) \ , \qquad (p-2|Dp \perp Dp) \ . \tag{2.17}$$

2.1.2 Fundamental string background

In the string frame, the metric and dilaton for the background created by a fundamental string are of the form of eqs. (2.1) and (2.2) for:

$$\gamma_1 = 3 \ , \qquad \gamma_2 = 0 \ , \qquad \gamma_3 = \frac{3}{2} \ , \qquad R^6 = 32\pi^2 (\alpha')^3 \, g_s^2 \, N \ . \tag{2.18}$$

In this case one gets from (2.12) that $d = 0$, which corresponds to the following intersection:

$$(0|F1 \perp Dp) \ . \tag{2.19}$$

2.1.3 M2-brane background

Our next example is the geometry created by an M2-brane in M-theory. In this case one has:

$$\gamma_1 = 2 \ , \qquad \gamma_2 = 1 \ , \qquad \gamma_3 = 0 \ , \qquad R^6 = 32\pi^2 l_P^6 \, N \ , \tag{2.20}$$

where l_P is the Planck length in eleven dimensions. The worldvolume actions to be considered in this case are those of the M2- and M5-branes. The action of the M2-brane is the sum of a Born-Infeld term (given by (2.9) with $\gamma_3 = 0$) and a Wess-Zumino term which is just the pullback to the worldvolume of the three-form potential of eleven-dimensional supergravity. This pullback vanishes for the static configurations studied here and, thus, we can apply safely to this case the analysis based on eq. (2.9). Moreover, the worldvolume dynamics of the M5-brane will be described by the so-called PST formalism [26]. The corresponding action contains a Born-Infeld part, together with other terms which involve a worldvolume gauge field and the pullbacks of the three- and six-form potentials of eleven-dimensional supergravity. These pullbacks vanish for the static configurations we

JHEP04(2006)037

are now analyzing. Furthermore, for the orthogonal intersections we are interested in, the worldvolume gauge field is zero, as it was in the case of D-branes. Taking these facts into account, one easily concludes that the PST action reduces for these configurations to the form displayed in eq. (2.9) and, therefore, the no-force condition (2.12) is also valid in this case.

By using the values of the γ_i coefficients written in eq. (2.20), eq. (2.12) becomes

$$d = \frac{p_2 - 2}{3} \ . \tag{2.21}$$

Taking $p_2 = 2, 5$ we get the following intersections:

$$(0|M2 \perp M2) \ , \qquad\qquad (1|M2 \perp M5) \ , \tag{2.22}$$

which are indeed the same as those obtained by studying the supersymmetric solutions of eleven-dimensional supergravity (see, for example [27]).

2.1.4 M5-brane background

The background corresponding to an M5-brane in eleven dimensional supergravity has

$$\gamma_1 = \frac{1}{2} \ , \qquad \gamma_2 = 1 \ , \qquad \gamma_3 = 0 \ , \qquad R^3 = \pi\, l_P^3\, N \ , \tag{2.23}$$

which leads to

$$d = \frac{2p_2 - 1}{3} \ . \tag{2.24}$$

For $p_2 = 5$ in the previous expression we get the intersection:

$$(3|M5 \perp M5) \ , \tag{2.25}$$

which again coincides with the one found in [27].

2.2 Fluctuations

In what follows we will assume that the condition (2.11) holds. This fact can be checked for all the particular supersymmetric intersections that will be analyzed below.

Let us now study the fluctuations around the $|\vec{z}| = L$ embedding. Without loss of generality we can take $z^1 = L$, $z^m = 0$ $(m > 1)$ as the unperturbed configuration and consider a fluctuation of the type:

$$z^1 = L + \chi^1 \ , \qquad z^m = \chi^m \ (m > 1) \ , \tag{2.26}$$

where the χ's are small. The dynamics of the fluctuations is determined by the Dirac-Born-Infeld lagrangian which, for the fluctuations of the transverse scalars we study in this section, reduces to $\mathcal{L} = -e^{-\phi}\sqrt{-\det g}$, where g is the induced metric on the worldvolume. By expanding this lagrangian and keeping up to second order terms, one can prove that:

$$\mathcal{L} \,=\, -\frac{1}{2}\,\rho^{p_2-d-1}\,\sqrt{\det\tilde{g}}\,\left[\frac{R^2}{\rho^2+L^2}\right]^{\gamma_2}\,\mathcal{G}^{ab}\,\partial_a\chi^m\,\partial_b\chi^m \ , \tag{2.27}$$

JHEP04(2006)037

where $\mathcal{G}^{ab}$ is the (inverse) of the metric (2.8). The equations of motion derived from this lagrangian are:

$$\partial_a \left[\frac{\rho^{p_2-d-1}\sqrt{\det \tilde{g}}}{(\rho^2+L^2)^{\gamma_2}}\, \mathcal{G}^{ab}\, \partial_b \chi \right] = 0 \ , \tag{2.28}$$

where we have dropped the index m in the χ's. Using the explicit form of the metric elements $\mathcal{G}^{ab}$, the above equation can be written as:

$$\frac{R^{2\gamma_1+2\gamma_2}}{(\rho^2+L^2)^{\gamma_1+\gamma_2}}\, \partial^\mu \partial_\mu \chi + \frac{1}{\rho^{p_2-d-1}}\, \partial_\rho \left(\rho^{p_2-d-1}\partial_\rho \chi\right) + \frac{1}{\rho^2}\, \nabla^i \nabla_i \chi = 0 \ , \tag{2.29}$$

where the index μ corresponds to the directions $x^\mu = (t, x^1, \ldots, x^d)$ and ∇_i is the covariant derivative on the (p_2-d-1)-sphere. In order to analyze this equation, let us separate variables as:

$$\chi = \xi(\rho)\, e^{ikx}\, Y^l(S^{p_2-d-1}) \ , \tag{2.30}$$

where the product kx is performed with the flat Minkowski metric and $Y^l(S^{p_2-d-1})$ are scalar spherical harmonics which satisfy:

$$\nabla^i \nabla_i\, Y^l(S^{p_2-d-1}) = -l(l+p_2-d-2)\; Y^l(S^{p_2-d-1}) \ . \tag{2.31}$$

If we redefine the variables as:

$$\varrho = \frac{\rho}{L} \ , \qquad \bar{M}^2 = -R^{2\gamma_1+2\gamma_2}\, L^{2-2\gamma_1-2\gamma_2}\, k^2 \ , \tag{2.32}$$

the differential equation becomes:

$$\partial_\varrho \left(\varrho^{p_2-d-1}\partial_\varrho\, \xi\right) + \left[\bar{M}^2\, \frac{\varrho^{p_2-d-1}}{(1+\varrho^2)^{\gamma_1+\gamma_2}} - l(l+p_2-d-2)\varrho^{p_2-d-3} \right] \xi = 0 \ . \tag{2.33}$$

In order to study the solutions of eq. (2.33), let us change variables in such a way that this equation can be written as a Schrödinger equation:

$$\partial_y^2\, \psi - V(y)\, \psi = 0 \ , \tag{2.34}$$

where V is some potential. The change of variables needed to convert eq. (2.33) into (2.34) is:

$$e^y = \varrho \ , \qquad \psi = \varrho^{\frac{p_2-d-2}{2}}\ \xi \ . \tag{2.35}$$

Notice that in this change of variables $\varrho \to \infty$ corresponds to $y \to \infty$, while the point $\varrho = 0$ is mapped into $y = -\infty$. Moreover, the resulting potential $V(y)$ takes the form:

$$V(y) = \left(l - 1 + \frac{p_2-d}{2} \right)^2 - \bar{M}^2\, \frac{e^{2y}}{(e^{2y}+1)^{\gamma_1+\gamma_2}} \ . \tag{2.36}$$

In figure 3 we have plotted the function $V(y)$. It is interesting to notice that, in these new variables, the problem of finding the mass spectrum can be rephrased as that of finding the values of $\bar{M}$ such that a zero-energy level for the potential (2.36) exists. Notice that the classically allowed region in the Schrödinger equation (2.34) corresponds to the values

JHEP04(2006)037

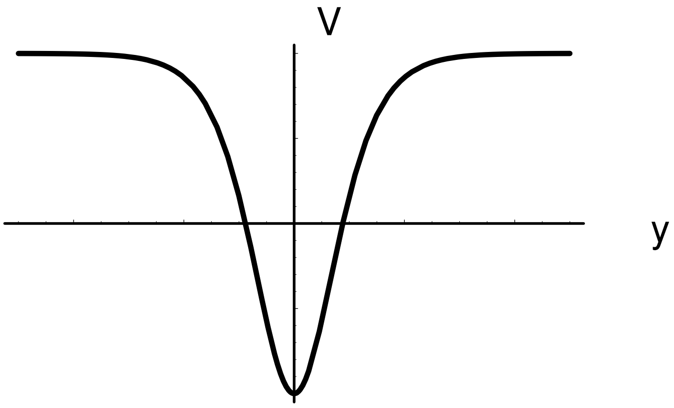

Figure 3: The Schrödinger potential $V(y)$ of eq. (2.36).

of y such that $V(y) \leq 0$. We would have a discrete spectrum if this region is of finite size or, equivalently, if the points $y = \pm\infty$ are not allowed classically. As, when $\gamma_1 + \gamma_2 > 1$, one has:

$$\lim_{y \to \pm\infty} V(y) = \left(l - 1 + \frac{p_2 - d}{2} \right)^2 , \tag{2.37}$$

we will have a discrete spectrum for all values of $l \in \mathbb{Z}_+$ if $p_2 - d > 2$. Notice that $p_2 - d \geq 2$ and when $p_2 - d = 2$ and $l = 0$ the turning points of the potential $V(y)$ are at $y = \pm\infty$. Moreover, $V(y)$ has a unique minimum at a value of y given by:

$$e^{2y_0} = \frac{1}{\gamma_1 + \gamma_2 - 1} . \tag{2.38}$$

2.3 The exactly solvable case

When $\gamma_1 + \gamma_2 = 2$, the differential equation for the fluctuation can be solved exactly in terms of a hypergeometric function (see appendix A). To prove this statement, let us change variables in eq. (2.33) as follows:

$$z = -\varrho^2 . \tag{2.39}$$

One can check that eq. (2.33) for $\gamma_1 + \gamma_2 = 2$ is converted into:

$$z(1-z)\frac{\partial^2 \xi}{\partial z^2} + \frac{p_2 - d}{2}(1-z)\frac{\partial \xi}{\partial z} + \left[\frac{l(l + p_2 - d - 2)}{4}(1 - z^{-1}) - \frac{\bar{M}^2}{4}(1-z)^{-1} \right] \xi = 0 , \tag{2.40}$$

which can be reduced to the hypergeometric differential equation. Indeed, let us define λ as:

$$\lambda \equiv \frac{-1 + \sqrt{1 + \bar{M}^2}}{2} . \tag{2.41}$$

Notice that eq. (2.41) can be easily inverted, namely:

$$\bar{M}^2 = 4\lambda(\lambda + 1) . \tag{2.42}$$

JHEP04(2006)037

Then, in terms of the original variable ϱ, the solution of eq. (2.40) that is regular when $\varrho \to 0$ is:

$$\xi(\varrho) = \varrho^l \, (\varrho^2 + 1)^{-\lambda} \, F\left(-\lambda, -\lambda + l - 1 + \frac{p_2 - d}{2}; l + \frac{p_2 - d}{2}; -\varrho^2\right) . \tag{2.43}$$

We also want that ξ vanishes when $\varrho \to \infty$. A way to ensure this is by imposing that

$$-\lambda + l - 1 + \frac{p_2 - d}{2} = -n \, , \qquad\qquad n = 0, 1, 2, \ldots \, . \tag{2.44}$$

When this condition is satisfied the hypergeometric function behaves as $(\varrho^2)^n$ when $\varrho \to \infty$ and $\xi \sim \varrho^{-(l+p_2-d-2)}$ in this limit. Notice that when $p_2 - d = 2$ the $l = 0$ mode does not vanish at large ϱ, in agreement with our general analysis based on the potential (2.36). By using the condition (2.44) in eq. (2.42), one gets:

$$\bar{M}^2 = 4\left(n + l - 1 + \frac{p_2 - d}{2}\right)\left(n + l + \frac{p_2 - d}{2}\right) . \tag{2.45}$$

Since in this case $\bar{M}^2 = -R^4 L^{-2} k^2$, one gets the following spectrum of possible values of $k^2 = -M^2$:

$$M^2 = \frac{4L^2}{R^4}\left(n + l + \frac{p_2 - d - 2}{2}\right)\left(n + l + \frac{p_2 - d}{2}\right) . \tag{2.46}$$

Let us look in detail which intersections satisfy the condition $\gamma_1 + \gamma_2 = 2$, needed to reduce the fluctuation equation to the hypergeometric one. From the list of intersections worked out in subsection 2.1, it is clear that this exactly solvable cases can only occur if the background is a Dp-brane. Actually, in this case one must have $\gamma_1 = \gamma_2 = 1$ (see eq. (2.14)), which only happens if $p = 3$. Thus, the list of exactly solvable intersections reduces to the following cases:

$$(3|D3 \perp D7) \, , \qquad (2|D3 \perp D5) \, , \qquad (1|D3 \perp D3) \, . \tag{2.47}$$

Notice that, for the three cases in (2.47), $p_2 = 2d + 1$ for $d = 3, 2, 1$. Therefore, if $dx^2_{1,d}$ denotes the line element for the flat Minkowski space in $d + 1$ dimensions, one can write the induced metric (2.8) as:

$$ds^2_I = \frac{\rho^2 + L^2}{R^2} \, dx^2_{1,d} + \frac{R^2}{\rho^2 + L^2} \, d\rho^2 + R^2 \, \frac{\rho^2}{\rho^2 + L^2} \, d\Omega^2_d \, . \tag{2.48}$$

Moreover, by using the relation between p_2 and d, one can rewrite the mass spectra (2.46) of scalar fluctuations for the intersections (2.47) as:

$$M_S = \frac{2L}{R^2} \, \sqrt{\left(n + l + \frac{d-1}{2}\right)\left(n + l + \frac{d+1}{2}\right)} \, . \tag{2.49}$$

In the three cases in (2.47) the background geometry is $AdS_5 \times S^5$. Moreover, one can see from (2.48) that the induced metric reduces in the UV limit $\rho \to \infty$ to that of a product space of the form $AdS_{d+2} \times S^d$. Indeed, the $(3|D3 \perp D7)$ intersection is the case

JHEP04(2006)037

extensively studied in ref. [7] and corresponds in the UV to an $AdS_5 \times S^3 \subset AdS_5 \times S^5$ embedding. In this case the D7-brane is a flavor brane for the $\mathcal{N} = 4$ gauge theory. The $(2|D3 \perp D5)$ intersection represents in the UV an $AdS_4 \times S^2$ defect in $AdS_5 \times S^5$. In the conformal limit $L = 0$ the corresponding defect CFT has been studied in detail in ref. [21] where, in particular, the fluctuation/operator dictionary was found. In section 3 we will extend these results to the case in which the brane separation L is different from zero and we will be able to find analytical expressions for the complete set of fluctuations. The $(1|D3 \perp D3)$ intersection corresponds in the UV to an $AdS_3 \times S^1$ defect in $AdS_5 \times S^5$ which is of codimension two in the gauge theory directions. The fluctuation spectra and the corresponding field theory dual for $L = 0$ have been analyzed in ref. [24]. In section 4 we will integrate analytically the differential equations for all the fluctuations of this $(1|D3 \perp D3)$ intersection when the D3-brane separation L is non-vanishing.

The $\varrho \to \infty$ limit is simply the high energy regime of the theory, where the mass of the quarks can be ignored and the theory becomes conformal. Therefore, the $\varrho \to \infty$ behaviour of the fluctuations should provide us information about the the conformal dimension Δ of the corresponding dual operators. Indeed, in the context of the AdS/CFT correspondence in $d + 1$ dimensions, it is well known that, if the fields are canonically normalized, the normalizable modes behave at infinity as $\rho^{-\Delta}$, whereas the non-normalizable ones should behave as $\rho^{\Delta-d-1}$. In the case in which the modes are not canonically normalized the behaviours of both types of modes are of the form $\rho^{-\Delta+\gamma}$ and $\rho^{\Delta-d-1+\gamma}$ for some γ. Clearly we can obtain the conformal dimension from the difference between the exponents. Let us apply this method to fluctuations which are given in terms of hypergeometric functions, as in eq. (2.43). Since for large ϱ the hypergeometric function behaves as:

$$F(a_1, a_2; b; -\varrho^2) \approx c_1\, \varrho^{-2a_1} + c_2\, \varrho^{-2a_2} \,, \qquad (\varrho \to \infty)\,, \tag{2.50}$$

one immediately gets:

$$\Delta = \frac{d+1}{2} + a_2 - a_1\,. \tag{2.51}$$

For the scalar fluctuations studied above, one has from the solution (2.43) that $a_1 = -\lambda$ and $a_2 = -\lambda + l + \frac{d-1}{2}$. By applying eq. (2.51) to this case, we get the following value for the dimension of the operator associated to the scalar fluctuations:

$$\Delta_S = l + d\,. \tag{2.52}$$

Notice that the quantization condition (2.44) selects precisely the normalizable modes, which behave at large ϱ as $\xi \sim \varrho^{-\Delta_S+1}$. Notice also that the modes that become constant at $\rho \to \infty$ correspond to operators with $\Delta_S = 1$ and, therefore, they should not be discarded. A glance at eq. (2.52) reveals that this situation only occurs when $d = 1$ (i.e. for the $AdS_3 \times S^1$ defect) and $l = 0$. This case will be studied in detail in section 4.

As the hypergeometric function $F(a_1, a_2; b; -\varrho^2)$ is symmetric under $a_1 \leftrightarrow a_2$, it is clear that the roles of a_1 and a_2 can also be exchanged in (2.51). If the resulting conformal dimension lies in the unitarity range $\Delta > 0$ (or $\Delta \geq 0$ if $d = 1$) we have a second branch of fluctuations. For the case at hand $\Delta = 1 - l$ and the unitarity condition requires

JHEP04(2006)037

generically that $l = 0$. This second branch is selected by imposing to the hypergeometric function (2.43) the truncation condition $\lambda = n$ for $n = 1, 2, \ldots$. The resulting spectrum is just $\bar{M}^2 = 4n(n+1)$, $n = 1, 2, \ldots$. In the rest of this paper this second branch of the fluctuations of the transverse scalars will not be considered further.

2.4 WKB quantization

The mapping to the Schrödinger equation we have performed in section 2.2 allows us to apply the semiclassical WKB approximation to compute the fluctuation spectrum. The WKB method has been very successful [29, 30] in the calculation of the glueball spectrum in the context of the gauge/gravity correspondence [31]. The starting point in this calculation is the WKB quantization rule:

$$\left(n + \frac{1}{2}\right)\pi = \int_{y_1}^{y_2} dy\, \sqrt{-V(y)}\ , \qquad n \geq 0\ , \tag{2.53}$$

where $n \in \mathbb{Z}$ and y_1, y_2 are the turning points of the potential ($V(y_1) = V(y_2) = 0$). To evaluate the right-hand side of (2.53) we expand it as a power series in $1/\bar{M}$ and keep the leading and subleading terms of this expansion. We obtain in this way the expression of $\bar{M}$ as a function of the principal quantum number n which is, in principle, reliable for large n, although in some cases it happens to give the exact result. Let us recall the outcome of this analysis for a general case, following [30]. With this purpose, let us come back to the original variable ϱ and suppose that we have a differential equation of the type:

$$\partial_\varrho\left(g(\varrho)\,\partial_\varrho\,\phi\right) + \left(\bar{M}^2\, q(\varrho) + p(\varrho)\right)\phi = 0\ , \tag{2.54}$$

where the functions g, q and p behave near $\varrho \approx 0, \infty$ as:

$$\begin{aligned} &g \approx g_1\varrho^{s_1}\ , \qquad q \approx q_1\varrho^{s_2}\ , \qquad p \approx p_1\varrho^{s_3}\ , \qquad \text{as } \varrho \to 0\ ,\\ &g \approx g_2\varrho^{r_1}\ , \qquad q \approx q_2\varrho^{r_2}\ , \qquad p \approx p_2\varrho^{r_3}\ , \qquad \text{as } \varrho \to \infty\ . \end{aligned} \tag{2.55}$$

The consistency of the WKB approximation requires that $s_2 - s_1 + 2$ and $r_1 - r_2 - 2$ be stricly positive numbers, whereas $s_3 - s_1 + 2$ and $r_1 - r_3 - 2$ can be either positive or zero [30]. In our case (eq. (2.33)), the functions $g(\varrho)$, $q(\varrho)$ and $p(\varrho)$ are:

$$g(\varrho) = \varrho^{p_2-d-1}\ , \qquad q(\varrho) = \frac{\varrho^{p_2-d-1}}{(1+\varrho^2)^{\gamma_1+\gamma_2}}\ , \qquad p(\varrho) = -l(l+p_2-d-2)\varrho^{p_2-d-3}\ . \tag{2.56}$$

From the behavior at $\rho \approx 0$ of the functions written above, we obtain:

$$\begin{aligned} &g_1 = 1\ , &&s_1 = p_2 - d - 1\ ,\\ &q_1 = 1\ , &&s_2 = p_2 - d - 1\ ,\\ &p_1 = -l(l+p_2-d-2)\ , &&s_3 = p_2 - d - 3\ . \end{aligned} \tag{2.57}$$

Notice that $s_2 - s_1 + 2 = 2$ and $s_3 - s_1 + 2 = 0$ and, thus, we are within the range of applicability of the WKB approximation. Moreover, from the behavior at $\rho \to \infty$ of the

JHEP04(2006)037

functions written in (2.56) we obtain:

$$\begin{aligned} g_2 &= 1 \, , & r_1 &= p_2 - d - 1 \, , \\ q_2 &= 1 \, , & r_2 &= p_2 - d - 1 - 2\gamma_1 - 2\gamma_2 \, , \\ p_2 &= -l(l + p_2 - d - 2) \, , & r_3 &= p_2 - d - 3 \, . \end{aligned} \tag{2.58}$$

Now $r_1 - r_2 - 2 = 2(\gamma_1 + \gamma_2 - 1)$ and $r_1 - r_3 - 2 = 0$ and we are also in the range of applicability of the WKB method if $\gamma_1 + \gamma_2 > 1$. Coming back to the general case, let us define [30] the quantities:

$$\alpha_1 = s_2 - s_1 + 2 \, , \qquad \beta_1 = r_1 - r_2 - 2 \, , \tag{2.59}$$

and (as $s_3 - s_1 + 2 = r_1 - r_3 - 2 = 0$, see [30]):

$$\alpha_2 = \sqrt{(s_1 - 1)^2 - 4\,\frac{p_1}{g_1}} \, , \qquad \beta_2 = \sqrt{(r_1 - 1)^2 - 4\,\frac{p_2}{g_2}} \, . \tag{2.60}$$

Then, the mass levels for large quantum number n can be written in terms of $\alpha_{1,2}$ and $\beta_{1,2}$ as [30]:

$$\bar{M}^2_{WKB} = \frac{\pi^2}{\zeta^2}\,(n+1)\left(n + \frac{\alpha_2}{\alpha_1} + \frac{\beta_2}{\beta_1}\right) , \tag{2.61}$$

where ζ is the following integral:

$$\zeta = \int_0^\infty d\varrho\,\sqrt{\frac{q(\varrho)}{g(\varrho)}} \, . \tag{2.62}$$

In our case $\alpha_{1,2}$ and $\beta_{1,2}$ are easily obtained from the coefficients written in (2.57) and (2.58), namely:

$$\alpha_1 = 2 \, , \qquad \beta_1 = 2(\gamma_1 + \gamma_2 - 1) \, , \qquad \alpha_2 = \beta_2 = 2l + p_2 - d - 2 \, . \tag{2.63}$$

Moreover, the integral ζ for our system is given by:

$$\zeta = \int_0^\infty \frac{d\varrho}{(1+\varrho^2)^{\frac{\gamma_1+\gamma_2}{2}}} = \frac{\sqrt{\pi}}{2}\,\frac{\Gamma\left(\frac{\gamma_1+\gamma_2-1}{2}\right)}{\Gamma\left(\frac{\gamma_1+\gamma_2}{2}\right)} \, , \tag{2.64}$$

and we get the following WKB formula for the masses:

$$\bar{M}_S^{WKB} = 2\sqrt{\pi}\,\frac{\Gamma\left(\frac{\gamma_1+\gamma_2}{2}\right)}{\Gamma\left(\frac{\gamma_1+\gamma_2-1}{2}\right)}\sqrt{(n+1)\left(n + \frac{\gamma_1+\gamma_2}{\gamma_1+\gamma_2-1}\left(l - 1 + \frac{p_2 - d}{2}\right)\right)} \, . \tag{2.65}$$

2.5 Numerical computation

The formula (2.65) for the masses can be checked numerically by means of the shooting technique. Notice that the behaviour for small ϱ of the fluctuation ξ (needed when this

JHEP04(2006)037

technique is applied) can be easily obtained. Indeed, let us try to find a solution of eq. (2.33) of the form:

$$\xi \sim \varrho^{\gamma} \ , \tag{2.66}$$

and let us neglect the term containing $\bar{M}^2$ of eq. (2.33). It is immediate to see that γ satisfies the equation:

$$\gamma^2 + (p_2 - d - 2)\gamma - l(l + p_2 - d - 2) = 0 \ , \tag{2.67}$$

whose roots are:

$$\gamma = l, -(l + p_2 - d - 2) \ . \tag{2.68}$$

Clearly, the solution regular at $\varrho = 0$ should correspond to the root $\gamma = l$. Then, we conclude that near $\varrho = 0$ one has:

$$\xi \sim \varrho^{l} \ , \qquad\qquad (\varrho \approx 0) \ . \tag{2.69}$$

In order to get the mass levels in the numerical calculation we have to match the $\varrho \approx 0$ behaviour (2.69) with the behaviour for large ϱ. The latter can be easily obtained by using the mapping written in (2.35) to the Schrödinger equation (2.34). Indeed, for $\varrho \to \infty$, or equivalently for $y \to \infty$, the potential $V(y)$ becomes asymptotically constant and the wave equation (2.34) can be trivially integrated. Let us call $V_* \equiv \lim_{y\to\infty} V(y)$, with $V_* > 0$. Then, the solutions of (2.34) are of the form $\psi \sim e^{\pm\sqrt{V_*}y}$ which, in terms of the original variable ϱ are simply $\psi \sim \varrho^{\pm\sqrt{V_*}}$. The actual value of V_* is given in eq. (2.37). Taking into account the relation (2.35) between ψ and ξ, we get that $\xi \sim \varrho^{\gamma}$ for $\varrho \to \infty$, where γ are exactly the two values written in eq. (2.68). The so-called S^l modes are characterized by the following UV behaviour:

$$\xi \sim \varrho^{-(l+p_2-d-2)} \ , \qquad\qquad (\varrho \to \infty) \ . \tag{2.70}$$

In the shooting technique one solves the differential equation for the fluctuations by imposing the behaviour (2.69) at $\varrho \approx 0$ and then one scans the values of $\bar{M}$ until the UV behaviour (2.70) is fine tuned. This occurs only for a discrete set of values of $\bar{M}$, which determines the mass spectrum we are looking for. The numerical values obtained in the different intersections and their comparison with the WKB mass formula (2.65) are studied in the appendices.

3. Fluctuations of the D3-D5 system

In this section we study in detail the complete set of fluctuations corresponding to the $(2|D3 \perp D5)$ intersection. The dynamics of the D5-brane probe in the $AdS_5 \times S^5$ background is governed by the Dirac-Born-Infeld action, which in this case reduces to

$$S = -\int d^6\xi \, \sqrt{-\det(g+F)} + \int d^6\xi \ P[C^{(4)}] \wedge F \ , \tag{3.1}$$

where g is the induced metric on the worldvolume, $P[\cdots]$ denotes the pullback of the form inside the brackets and, for convenience, we are taking the D5-brane tension equal to one.

JHEP04(2006)037

In (3.1) F is the two-form corresponding to the worldvolume field strength, whose one-form potential will be denoted by A ($F = dA$).

Let us now find the action for the complete set of quadratic fluctuations around the static configuration in which the two branes are at a distance L. Recall that in this embedding the worldvolume metric in the UV is $AdS_4 \times S^2$. As in section 2, let us denote by χ the scalars transverse to both types of branes and let us assume that the probe is extended along x^1 and x^2. We will call simply X to the coordinate x^3. By expanding up to second order the action (3.1), one gets the following lagrangian for the fluctuations:

$$\mathcal{L} = -\rho^2 \sqrt{\tilde{g}} \left[\frac{1}{2} \frac{R^2}{\rho^2 + L^2} \mathcal{G}^{ab} \partial_a \chi \partial_b \chi + \frac{1}{2} \frac{\rho^2 + L^2}{R^2} \mathcal{G}^{ab} \partial_a X \partial_b X + \frac{1}{4} F_{ab} F^{ab} \right] - \\ - 2 \frac{\rho}{R^4} (\rho^2 + L^2) X \epsilon^{ij} F_{ij} \ , \tag{3.2}$$

where i, j are indices of the two-sphere of the worldvolume, $\epsilon^{ij} = \pm 1$ and $\mathcal{G}_{ab}$ is the induced metric for the static configuration, i.e. the metric displayed in eq. (2.48) for $d = 2$.

The equation of motion of the scalar χ is just (2.28) for $p_2 = 5$, $d = 2$ and $\gamma_2 = 1$. As shown in subsection 2.3 this equation can be solved exactly in terms of the hypergeometric function (2.43). Upon imposing the quantization condition (2.44) we obtain a tower of normalizable modes, which we will denote by S^l, given by:

$$\xi_S(\varrho) = \varrho^l \, (\varrho^2 + 1)^{-n-l-\frac{1}{2}} \, F\left(-n - l - \frac{1}{2}, -n; l + \frac{3}{2}; -\varrho^2 \right), \qquad (l, n \geq 0) \, . \tag{3.3}$$

Recall from (2.49) and (2.52) that the associated mass $M_S(n, l)$ and conformal dimension Δ_S for these scalar modes are given by:

$$M_S(n, l) = \frac{2L}{R^2} \sqrt{\left(n + l + \frac{1}{2} \right) \left(n + l + \frac{3}{2} \right)} \ , \qquad \Delta_S = l + 2 \ . \tag{3.4}$$

Notice that the value found here for Δ_S is in agreement with the result of ref. [21].

As shown in (3.2), the scalar X is coupled to the components F_{ij} of the gauge field strength along the two-sphere. The equation of motion of X derived from the lagrangian (3.2) is:

$$R^2 \, \partial_a \left[\rho^2 \sqrt{\tilde{g}} \, (\rho^2 + L^2) \, \mathcal{G}^{ab} \partial_b X \right] - 2\rho \, (\rho^2 + L^2) \, \epsilon^{ij} F_{ij} = 0 \ . \tag{3.5}$$

Moreover, the equation of motion of the gauge field is:

$$R^4 \, \partial_a \left[\rho^2 \sqrt{\tilde{g}} \, F^{ab} \right] - 4\rho \, (\rho^2 + L^2) \, \epsilon^{bj} \, \partial_j X = 0 \ , \tag{3.6}$$

where ϵ^{bj} is zero unless b is an index along the two-sphere.

Let us now see how the equations of motion (3.5) and (3.6) can be decoupled and, subsequently, integrated in analytic form. With this purpose in mind, let us see how one can obtain vector spherical harmonics for the two-sphere from the scalar harmonics Y^l.

JHEP04(2006)037

Clearly, from a scalar harmonic in S^2 we can can construct a vector by simply taking the derivative with respect to the coordinates of the two-sphere, namely:

$$Y_i^l(S^2) \equiv \nabla_i \, Y^l(S^2) \; . \tag{3.7}$$

One can check from (2.31) that these functions satisfy:

$$\begin{aligned} &\nabla^i \, Y_i^l = \frac{1}{\sqrt{\tilde g}} \, \partial_i \Big[\sqrt{\tilde g} \, \tilde g^{ij} \, Y_j^l \Big] = -l(l+1) \, Y^l \; , \\ &\epsilon^{ij} \, \partial_i \, Y_j^l = 0 \; . \end{aligned} \tag{3.8}$$

Alternatively, we can take the Hogde dual in the sphere and define a new vector harmonic function $\hat Y_i^l(S^2)$ as:

$$\hat Y_i^l(S^2) \; \equiv \; \frac{1}{\sqrt{\tilde g}} \, \tilde g_{ij} \, \epsilon^{jk} \, \nabla_k \, Y^l(S^2) \; . \tag{3.9}$$

The $\hat Y_i^l$ vector harmonics satisfy:

$$\begin{aligned} \nabla^i \, \hat Y_i^l \; &= \; 0 \; , \\ \epsilon^{ij} \, \partial_i \, \hat Y_j^l \; &= \; l(l+1) \, \sqrt{\tilde g} \; Y^l \; . \end{aligned} \tag{3.10}$$

Let us analyze the different types of modes, in analogy with the D3-D7 case in [7].

3.1 Type I modes

We are going to study first the modes which involve the scalar field X and the components A_i of the gauge field along the two-sphere directions. Generically, the equations of motion couple A_i to the other gauge field components A_μ and A_ρ. However, due to the property $\nabla^i \, \hat Y_i^l = 0$ (see eq. (3.10)), if A_i is proportional to $\hat Y_i^l$ it does not mix with other components of the gauge field, although it mixes with the scalar X. Accordingly, let us take the ansatz:

$$A_\mu = 0 \; , \qquad A_\rho = 0 \; , \qquad A_i = \phi(x,\rho) \, \hat Y_i^l(S^2) \; , \tag{3.11}$$

while we represent X as:

$$X = \Lambda(x,\rho) \, Y^l(S^2) \; . \tag{3.12}$$

Taking into account that

$$\frac{1}{2} \, \epsilon^{ij} \, F_{ij} = l(l+1) \, \sqrt{\tilde g} \, \phi \, Y^l \; , \tag{3.13}$$

one can prove that the equation of motion of X (eq. (3.5)) becomes:

$$\begin{aligned} R^4 \, \rho^2 \, \partial_\mu \partial^\mu \Lambda + \partial_\rho \Big[\rho^2 \, (\rho^2 + L^2)^2 \, \partial_\rho \, \Lambda \Big] - \\ - l(l+1) \, (\rho^2 + L^2)^2 \, \Lambda - 4l(l+1) \, \rho \, (\rho^2 + L^2) \phi = 0 \; . \end{aligned} \tag{3.14}$$

It can be easily verified that the equations for A_μ and A_ρ are automatically satisfied as a consequence of the relation $\nabla^i \, \hat Y_i^l = 0$. Moreover, for $l \neq 0$ the equation of motion (3.6) for the gauge field components along S^2 reduces to:

$$R^4 \, \partial_\mu \partial^\mu \phi + \partial_\rho \Big[\, (\rho^2 + L^2)^2 \, \partial_\rho \, \phi \Big] - l(l+1) \, \frac{(\rho^2 + L^2)^2}{\rho^2} \, \phi \, - 4\rho \, (\rho^2 + L^2) \Lambda = 0 \; . \tag{3.15}$$

JHEP04(2006)037

In order to decouple this system of equations, let us follow closely the steps of ref. [21]. First, we redefine the scalar field Λ as follows:

$$V = \rho \Lambda \ . \tag{3.16}$$

The system of equations which results after this redefinition can be decoupled by simply taking suitable linear combinations of the unknown functions V and ϕ. This decoupling procedure was used in ref. [21] for the conformal case $L = 0$ and, remarkably, it also works for the case in which the brane separation does not vanish. In appendix B we will apply this method to decouple the fluctuations of the type written in eqs. (3.11) and (3.12) for the more general $(p-1|Dp \perp D(p+2))$ intersection. Here we just need the decoupled functions, which are:

$$\begin{aligned} Z^+ &= V + l\phi \ , \\ Z^- &= V - (l+1)\,\phi \ . \end{aligned} \tag{3.17}$$

It is interesting at this point to notice that the Z^- modes only exist for $l \geq 1$ while the Z^+ modes make sense for $l \geq 0$. Indeed, the vector harmonic $\hat{Y}_i^l$ vanishes for $l = 0$, since it is the derivative of a constant function (see eq. (3.7)). Then, it follows from (3.11) that the vector field vanishes for these $l = 0$ modes and, as a result, the $l = 0$ mode of the scalar field is uncoupled. It is clear from (3.17) that this $l = 0$ mode of the field X is just Z^+ for vanishing l. As we have just mentioned, for $l > 0$ the equations for Z^+ and Z^- are decoupled. These equations can be obtained by substituting the definition (3.17) into eqs. (3.14) and (3.15). In order to get the corresponding spectra, let us adopt a plane wave ansatz for $Z^\pm$, namely:

$$Z^\pm = e^{ikx}\,\xi^\pm(\rho) \ . \tag{3.18}$$

Moreover, let us define the reduced variables ϱ and $\bar{M}$ as in eq. (2.32), namely $\varrho = \rho/L$ and $\bar{M} = -R^4 L^{-2} k^2$. Furthermore, we define λ as in eq. (2.41). We will consider separately the Z^+ and Z^- equations.

3.1.1 Z^+ spectra

By combining appropriately eqs. (3.14) and (3.15) one can show that the equation for ξ^+ is indeed decoupled and given by:

$$\frac{1}{1+\varrho^2}\,\partial_\varrho\Big[(1+\varrho^2)^2\,\partial_\varrho\,\xi^+\Big] + \left[\frac{\bar{M}^2}{1+\varrho^2} - (l+1)\left(l+4+\frac{l}{\varrho^2}\right)\right]\xi^+ = 0 \ . \tag{3.19}$$

Remarkably, eq. (3.19) can be analytically solved in terms of a hypergeometric function. Actually, by using the change of variables of appendix A, one can show that the solution of (3.19) which is regular at $\varrho = 0$ is:

$$\xi^+(\varrho) = \varrho^{1+l}\,(1+\varrho^2)^{-1-\lambda}\;F\left(-\lambda-1, l+\frac{3}{2}-\lambda; l+\frac{3}{2}; -\varrho^2\right) \ . \tag{3.20}$$

JHEP04(2006)037

By using eq. (2.50) one can show that, for large values of the ϱ coordinate, the function ξ^+ written in (3.20) behaves as:

$$\xi^+(\varrho) \sim c_1 \varrho^{l+1} + c_2 \varrho^{-l-4} , \qquad (\varrho \to \infty) . \tag{3.21}$$

Clearly, the only normalizable solutions are those for which $c_1 = 0$. This regularity condition at $\varrho = \infty$ can be enforced by means of the following quantization condition:

$$l + \frac{3}{2} - \lambda = -n , \qquad n = 0, 1, 2, \ldots . \tag{3.22}$$

The ξ^+ fluctuations for which (3.22) holds will be referred to as I_+^l modes. Their analytical expression is given by:

$$\xi_{I_+}(\varrho) = \varrho^{1+l} (\varrho^2 + 1)^{-n-l-\frac{5}{2}} F\left(-n - l - \frac{5}{2}, -n; l + \frac{3}{2}; -\varrho^2\right), \qquad (l, n \geq 0) , \tag{3.23}$$

and the corresponding energy levels are:

$$M_{I_+}(n, l) = \frac{2L}{R^2} \sqrt{\left(n + l + \frac{3}{2}\right)\left(n + l + \frac{5}{2}\right)} . \tag{3.24}$$

By using the general expression (2.51) one reaches the conclusion that the conformal dimensions of the operators dual to the I_+^l modes are:

$$\Delta_{I_+} = l + 4 , \tag{3.25}$$

which agrees with the values found in ref. [21].

3.1.2 Z^- spectra

The equation for ξ^- can be shown to be:

$$\frac{1}{1+\varrho^2} \partial_\varrho \Big[(1+\varrho^2)^2 \partial_\varrho \xi^-\Big] + \left[\frac{\bar{M}^2}{1+\varrho^2} - l\left(l - 3 + \frac{l+1}{\varrho^2}\right)\right] \xi^- = 0 , \tag{3.26}$$

which again can be solved in terms of the hypergeometric function. The solution regular at $\varrho = 0$ is:

$$\xi^-(\varrho) = \varrho^{1+l} (\varrho^2 + 1)^{-1-\lambda} F\Big(-\lambda + 1, l - \frac{1}{2} - \lambda; l + \frac{3}{2}; -\varrho^2\Big) . \tag{3.27}$$

It is easy to verify by using eq. (2.50) that $\xi^-(\varrho)$ has two possible behaviours at $\varrho \to \infty$, namely ϱ^{-l}, ϱ^{l-3}, where $l \geq 1$. The former corresponds to a normalizable mode with conformal dimension $\Delta = l$, while the latter is associated to operators with $\Delta = 3 - l$. Notice that the existence of these two branches is in agreement with the results of ref. [21].

Let us consider first the branch with $\Delta = l$, which we will refer to as I_-^l fluctuations. One can select these fluctuations by imposing the following quantization condition:

$$l - \frac{1}{2} - \lambda = -n , \qquad n = 0, 1, 2, \ldots . \tag{3.28}$$

JHEP04(2006)037

The corresponding functions $\xi^-(\varrho)$ are:

$$\xi_{I_-}(\varrho) = \varrho^{1+l}\,(\varrho^2+1)^{\,-n-l-\frac{1}{2}}\;F\Big(-n-l+\frac{3}{2},-n;l+\frac{3}{2};-\varrho^2\Big)\;, \qquad (l\geq 1, n\geq 0)\;, \tag{3.29}$$

and the mass spectrum and conformal dimension are:

$$M_{I_-}(n,l) = \frac{2L}{R^2}\sqrt{\Big(n+l-\frac{1}{2}\Big)\Big(n+l+\frac{1}{2}\Big)}\;, \qquad \Delta_{I_-} = l\;. \tag{3.30}$$

One can check that, indeed, the solution (3.29) behaves as ϱ^{-l} at $\varrho\rightarrow\infty$ and, therefore, the associated operator in the conformal limit has $\Delta = l$ as it should.

One can select the branch with $\Delta = 3-l$ by requiring that:

$$-\lambda + 1 = -n\;, \qquad n = 0,1,2,\dots\;. \tag{3.31}$$

The corresponding functions are:

$$\xi_{\tilde{I}_-}(\varrho) = \varrho^{1+l}\,(\varrho^2+1)^{\,-2-n}\;F\left(-n, l-n-\frac{3}{2}; l+\frac{3}{2}; -\varrho^2\right)\;. \tag{3.32}$$

Notice that the condition $\Delta = 3-l > 0$ is only fulfilled for two possible values of l, namely $l = 1,2$. We will refer to this branch of solutions as $\tilde{I}^l_-$ fluctuations. Their mass spectrum is independent of l, as follows from eq. (3.31). Thus, one has:

$$M_{\tilde{I}_-}(n,l) = \frac{2L}{R^2}\sqrt{\Big(n+1\Big)\Big(n+2\Big)}\;, \qquad \Delta_{\tilde{I}_-} = 3-l\;, \qquad (l=1,2)\;. \tag{3.33}$$

3.2 Type II modes

Let us consider a configuration with $X = 0$ and with the following ansatz for the gauge fields:

$$A_\mu = \phi_\mu(x,\rho)\,Y^l(S^2)\;, \qquad A_\rho = 0\;, \qquad A_i = 0\;, \tag{3.34}$$

with

$$\partial^\mu \phi_\mu = 0\;. \tag{3.35}$$

Due to this last condition one can check that the equations for A_ρ and A_i are satisfied, while the equation for A_μ yields:

$$\partial_\rho(\rho^2\partial_\rho\phi_\nu) + R^4\,\frac{\rho^2}{(\rho^2+L^2)^2}\,\partial_\mu\,\partial^\mu\phi_\nu - l(l+1)\,\phi_\nu = 0\;. \tag{3.36}$$

Expanding in a plane wave basis we get exactly the same equation as in the scalar fluctuations. Actually, let us represent ϕ_μ as:

$$\phi_\mu = \xi_\mu\,e^{ikx}\,\chi(\rho)\;, \tag{3.37}$$

where $k^\mu\,\xi_\mu = 0$. The equation for χ is:

$$\frac{1}{\varrho^2}\,\partial_\varrho\Big(\varrho^2\partial_\varrho\chi\Big) + \left[\frac{\bar{M}^2}{(1+\varrho^2)^2} - \frac{l(l+1)}{\varrho^2}\right]\chi = 0\;, \tag{3.38}$$

JHEP04(2006)037

where we have already introduced the reduced quantities ϱ and $\bar{M}$. This equation is identical to the one corresponding to the transverse scalars. Thus, the spectra are the same in both cases. These fluctuations correspond to an operator with conformal weight $\Delta_{II} = l + 2$, in agreement with ref. [21].

3.3 Type III modes

These modes have $X = 0$ and the following form for the gauge field:

$$A_\mu = 0 \;, \qquad A_\rho = \phi(x,\rho)\, Y^l(S^2) \;, \qquad A_i = \tilde{\phi}(x,\rho)\, Y^l_i(S^2) \;. \tag{3.39}$$

Notice that for the gauge field potential written above the field strength components F_{ij} along the two-sphere vanish, which ensures that the equation of motion of X is satisfied for $X = 0$. The non-vanishing components of the gauge field strenght are:

$$F_{\mu i} = \partial_\mu \tilde{\phi}\, Y^l_i \;, \qquad F_{\mu\rho} = \partial_\mu\, \phi\, Y^l \;, \qquad F_{\rho i} = (\partial_\rho \tilde{\phi} - \phi)\, Y^l_i \;. \tag{3.40}$$

The equation for A_ρ becomes:

$$R^4\, \rho^2\, \partial_\mu\, \partial^\mu \phi - l(l+1)\, (\rho^2 + L^2)^2\, (\phi - \partial_\rho \tilde{\phi}) = 0 \;, \tag{3.41}$$

while the equation for A_i is:

$$R^4\, \partial_\mu\, \partial^\mu \tilde{\phi} + \partial_\rho \Big[(\rho^2 + L^2)^2\, (\partial_\rho \tilde{\phi} - \phi) \Big] = 0 \;. \tag{3.42}$$

Moreover, the equation for A_μ can be written as:

$$\partial_\mu \Big[l(l+1) \tilde{\phi} - \partial_\rho (\rho^2 \phi) \Big] = 0 \;. \tag{3.43}$$

Expanding ϕ and $\tilde{\phi}$ in a plane wave basis we can get rid of the x^μ derivative and we can write the following relation between $\tilde{\phi}$ and ϕ:

$$l(l+1) \tilde{\phi} = \partial_\rho (\rho^2 \phi) \;. \tag{3.44}$$

For $l \neq 0$, one can use this relation to eliminate $\tilde{\phi}$ in favor of ϕ. The equation of motion of A_ρ becomes:

$$\partial^2_\rho\, (\rho^2\, \phi) - l(l+1)\, \phi + \frac{R^4\, \rho^2}{(\rho^2 + L^2)^2}\, \partial_\mu\, \partial^\mu \phi = 0 \;, \tag{3.45}$$

which, again, can be solved in terms of hypergeometric functions. The equation of motion of A_i is just equivalent to the above equation. To solve this equation, let us write

$$\phi(x, \rho) = e^{ikx}\, \zeta(\rho) \;. \tag{3.46}$$

Then, in terms of the reduced variable ϱ the equation for ζ becomes

$$\frac{1}{\varrho^2}\, \partial^2_\varrho \Big(\varrho^2 \zeta \Big) + \left[\frac{\bar{M}^2}{(1 + \varrho^2)^2} - \frac{l(l+1)}{\varrho^2} \right] \zeta = 0 \;, \tag{3.47}$$

JHEP04(2006)037

where $\bar{M}$ is the same as for the type I modes. Eq. (3.47) can be solved in terms of the hypergeometric function as:

$$\zeta(\varrho) = \varrho^{l-1}\,(\varrho^2+1)^{-\lambda}\;F\left(-\lambda, l+\frac{1}{2}-\lambda; l+\frac{3}{2}; -\varrho^2\right). \tag{3.48}$$

The quantization condition and the energy levels are just the same as for the transverse scalars and the type II modes. At $\rho \to \infty$, $\zeta \sim \rho^{-l-2}$. These fluctuations correspond to a field with conformal weight $\Delta_{III} = l+2$, as predicted in [21].

3.4 Fluctuation/operator correspondence

Let us recall the array corresponding to the D3-D5 intersection:

$$\begin{array}{rccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ D3: & \times & \times & \times & \_ & \_ & \_ & \_ & \_ & \_ \\ D5: & \times & \times & \_ & \times & \times & \times & \_ & \_ & \_ \end{array} \tag{3.49}$$

Before adding the D5-brane we have a SO(6) R-symmetry which corresponds to the rotation in the 456789 directions. The D5-brane breaks this SO(6) to $\mathrm{SU}(2)_H \times \mathrm{SU}(2)_V$, where the $\mathrm{SU}(2)_H$ corresponds to rotations in the 456 directions (which are along the D5-brane worldvolume) and the $\mathrm{SU}(2)_V$ is generated by rotations in the 789 subspace (which are the directions orthogonal to both types of branes).

Let us recall how the $\mathcal{N}=4$, $d=4$ gauge multiplet decomposes under the $\mathcal{N}=4$, $d=3$ supersymmetry. As it is well-known, the $\mathcal{N}=4$ gauge multiplet in four dimensions contains a vector A_μ (which has components along the four coordinates of the D3-brane worldvolume), six real scalars X^i (corresponding to the directions orthogonal to the D3-brane worldvolume) and four complex Weyl spinors λ^a. All these fields are in the adjoint representation of the gauge group. The $d=4$ vector field A_μ gives rise to a $d=3$ gauge field A_k and to a scalar field A_3. Both types of fields are singlets with respect to the $\mathrm{SU}(2)_H \times \mathrm{SU}(2)_V$ symmetry. Moreover, the adjoint scalars can be arranged in two sets as:

$$X_H = (X^4, X^5, X^6)\,, \qquad X_V = (X^7, X^8, X^9)\,. \tag{3.50}$$

Clearly, X_H transforms in the $(\mathbf{3},\mathbf{1})$ representation of $\mathrm{SU}(2)_H \times \mathrm{SU}(2)_V$ while X_V does it in the $(\mathbf{1},\mathbf{3})$. Finally, the spinors transform in the $(\mathbf{2},\mathbf{2})$ representation and will be denoted by λ^{im}. In addition to the bulk fields, the 3-5 strings introduce a $d=3$ complex hypermultiplet in the fundamental representation of the gauge field, whose components will be denoted by (q^m, ψ^i). The bosonic components q^m of this hypermultiplet transform in the $(\mathbf{2},\mathbf{1})$ representation, whereas the fermionic ones ψ^i are in the $(\mathbf{1},\mathbf{2})$ of $\mathrm{SU}(2)_H \times \mathrm{SU}(2)_V$. The dimensions and quantum numbers of the different fields just discussed are summarized in table 1.

Let us now determine the quantum numbers of the different fluctuations of the D3-D5 system. We will denote by S^l the scalar fluctuations and by I^l_+ and I^l_- the two types of vector fluctuations of type I. The I^l_- fluctuations which we will consider from now on are those corresponding to the operator of dimension $\Delta = l$. Moreover, the modes of

JHEP04(2006)037

Field	Δ	$\mathrm{SU}(2)_H$	$\mathrm{SU}(2)_V$
A_k	1	0	0
A_3	1	0	0
X_H	1	1	0
X_V	1	0	1
λ^{im}	3/2	1/2	1/2
q^m	1/2	1/2	0
ψ^i	1	0	1/2

Table 1: Quantum numbers and dimensions of the fields of the D3-D5 intersection.

Mode	Δ	$\mathrm{SU}(2)_H$	$\mathrm{SU}(2)_V$
S^l	$l+2$	$l \geq 0$	1
I_+^l	$l+4$	$l \geq 0$	0
I_-^l	l	$l \geq 1$	0
V^l	$l+2$	$l \geq 0$	0

Table 2: Quantum numbers and dimensions of the modes of the D3-D5 intersection.

type II and III correspond to fluctuations of the vector gauge field in AdS_4 and will be denoted collectively by V^l. In all cases, l corresponds to the quantum number of the spherical harmonics in the 456 directions and, thus, it can be identified with the isospin of the $\mathrm{SU}(2)_H$ representation. Moreover, it is clear that the scalar modes are fluctuations in the 789 directions and therefore are in the vector representation of $\mathrm{SU}(2)_V$, while the other fluctuations are singlets under $\mathrm{SU}(2)_V$. With all this data and with the values of the dimensions determined previously, we can fill the values displayed in table 2.

Let us now recall our results for the mass spectra. The mass of the scalar fluctuations $M_S(n,l)$ is given in eq. (3.4). The masses of the other modes are given in terms of $M_S(n,l)$ as:

$$M_{I_+}(n,l) = M_S(n,l+1) \ , \quad M_{I_-}(n,l) = M_S(n,l-1) \ , \quad M_V(n,l) = M_S(n,l) \ . \tag{3.51}$$

Let us now match, following ref. [21], the fluctuation modes with composite operators of the $\mathcal{N}=4$, $d=3$ defect theory by looking at the dimensions and $\mathrm{SU}(2)_H \times \mathrm{SU}(2)_V$ quantum numbers of these two types of objects. Let us consider first the fluctuation mode with the lowest dimension, which according to our previous results is I_-^1. This mode is a triplet of $\mathrm{SU}(2)_H$ and a singlet of $\mathrm{SU}(2)_V$ and has $\Delta = 1$. There is only one operator with these characteristics. Indeed, let us define the following operator:

$$\mathcal{C}^I \equiv \bar{q}^m \sigma^I_{mn}\, q^n \ , \tag{3.52}$$

where the σ^I are Pauli matrices. This operator has clearly the same dimension and $\mathrm{SU}(2)_H \times \mathrm{SU}(2)_V$ quantum numbers as the mode I_-^1. Therefore, we have the identification [21]:

$$I_-^1 \sim \mathcal{C}^I \ . \tag{3.53}$$

JHEP04(2006)037

Moreover, by acting with the supersymmetry generators we can obtain the other operators in the same multiplet as $\mathcal{C}^I$. The bosonic ones are [21]:

$$\begin{aligned}\mathcal{E}^A &= \bar{\psi}^i\,\sigma^A_{ij}\,\psi^j + 2\bar{q}^m X_V^{Aa} T^a q^m \;,\\ J_B^k &= i\bar{q}^m D^k q^m - i(D^k q^m)^\dagger\, q^m + \bar{\psi}^i\,\rho^k \psi^i \;,\end{aligned} \tag{3.54}$$

where the T^a are the matrices of the gauge group and the ρ^k are Dirac matrices in $d=3$. Notice that these two operators have dimension $\Delta=2$. Moreover, $\mathcal{E}^A$ transforms in the $(\mathbf{1},\mathbf{3})$ representation of $\mathrm{SU}(2)_H\times\mathrm{SU}(2)_V$, whereas J_B^k is a $\mathrm{SU}(2)_H\times\mathrm{SU}(2)_V$ singlet. It is straightforward to find the modes that have these same quantum numbers and dimension. Indeed, one gets that:

$$S^0\sim\mathcal{E}^A\;,\qquad V^0\sim J_B^k\;. \tag{3.55}$$

Notice that our mass spectrum is consistent with these identifications, since according to eq. (3.51), the fluctuations I^1_-, S^0 and V^0 have all the same mass spectrum, namely $M_s(n,0)$.

Let us next consider the modes corresponding to higher values of l. Following ref. [21] we define the operator

$$\mathcal{C}_l^{I_0\cdots I_l}\equiv\mathcal{C}^{(I_0}X_H^{I_1}\cdots X_H^{I_l)}\;, \tag{3.56}$$

where the parentheses stand for the traceless symmetrization of the indices. The operator $\mathcal{C}_l$ has dimension $\Delta=l+1$, is a singlet of $\mathrm{SU}(2)_V$ and transforms in the spin $l+1$ representation of $\mathrm{SU}(2)_H$. It is thus a natural candidate to be identified with the mode I^{l+1}_-, namely:

$$I^{l+1}_-\sim\mathcal{C}_l^{I_0\cdots I_l}\;. \tag{3.57}$$

The mode I^0_+ has been identified in [21] with a four-supercharge descendant of the second-floor chiral primary $\mathcal{C}_1^{IJ}$. Notice that our mass spectra supports this identification since $M_{I_+}(n,0)=M_{I_-}(n,2)$. Actually, our results are consistent with having the modes I^{l+1}_-, S^l, V^l and I^{l-1}_+ in the same massive supermultiplet for $l\geq1$ and with the identification of this supermultiplet with the one obtained from the chiral primary $\mathcal{C}_l$, i.e.:

$$(I^{l+1}_-,S^l,V^l,I^{l-1}_+)\sim(\mathcal{C}_l,\ldots)\;,\qquad(l\geq1)\;. \tag{3.58}$$

As a check notice that the four modes on the left-hand side of eq. (3.58) have the same mass spectrum, namely $M_S(n,l)$. Moreover, $\Delta(S^l)=\Delta(V^l)=\Delta(I^{l+1}_-)+1$ and $\Delta(I^{l-1}_+)=\Delta(I^{l+1}_-)+2$, which is in agreement with the fact that the supercharge has dimension 1/2.

4. Fluctuations of the D3-D3 system

Let us analyze in this section the modes of the $(1|D3\perp D3)$ intersection. In the probe approximation we are considering the equations of motion of these fluctuation modes are obtained from the Dirac-Born-Infeld action of a D3-brane in the $AdS_5\times S^5$ background. This action is given by:

$$S=-\int d^4\xi\,\sqrt{-\det(g+F)}+\int d^4\xi\;\;P[C^{(4)}\,]\;. \tag{4.1}$$

JHEP04(2006)037

We want to expand the action (4.1) around the static configuration in which the two branes are separated a distance L. Recall from section 2 that the induced metric on the worldvolume of the D3-brane probe of such a configuration is just the one written in eq. (2.48) for $d = 1$, which reduces to $AdS_3 \times S^1$ in the UV limit. As in section 2, let us denote by χ the scalars transverse to both types of branes. In this case the defect created by the probe has codimension two in the Minkowski directions of $AdS_5 \times S^5$. Let us assume that the D3-brane probe is extended along the Minkowski coordinate x^1 and let us define:

$$\lambda_1 = x^2 \;, \qquad \lambda_2 = x^3 \;. \tag{4.2}$$

With these notations, the lagrangian for the quadratic fluctuations can be readily obtained from (4.1):

$$\begin{aligned}\mathcal{L} = &-\frac{\rho}{2}\,\frac{R^2}{\rho^2+L^2}\,\mathcal{G}^{ab}\partial_a\chi\partial_b\chi - \frac{\rho}{2}\,\frac{\rho^2+L^2}{R^2}\,\mathcal{G}^{ab}\partial_a\lambda_i\partial_b\lambda_i - \frac{\rho}{4}\,F_{ab}F^{ab} + \\ &+\frac{(\rho^2+L^2)^2}{R^4}\,\epsilon^{ij}\partial_\rho\lambda_i\,\partial_\varphi\lambda_j \;,\end{aligned} \tag{4.3}$$

where $i, j = 1, 2$ and $\mathcal{G}_{ab}$ is the metric (2.48) for $d = 1$. The equation of motion of χ derived from (4.3) is just the one studied in section 2, namely (2.28), for $p_2 = 3$, $d = 1$ and $\gamma_2 = 1$. Separating variables as in (2.30) we arrive at an equation which can be analytically solved in terms of hypergeometric functions. The corresponding solution has been written in eq. (2.43). After imposing the truncation condition (2.44) we obtain the so-called S^l modes, whose explicit expression is:

$$\xi_S(\varrho) = \varrho^l\,(\varrho^2+1)^{-n-l}\,F(-n-l,-n;l+1;-\varrho^2) \;. \tag{4.4}$$

Notice that in this case the harmonics are just exponentials of the type $e^{il\varphi}$, where φ is just the angular coordinate of the S^1 circle. Then, the quantum number l can take also negative values. The modes written in (4.4) are those which are regular at $\rho = 0$ for non-negative l. When $l < 0$ one can get regular modes at the origin by using the second solution of the hypergeometric function. The result is just (4.4) with l changed by $-l$. However, since the scalar field χ whose fluctuation we are analyzing is real, changing l by $-l$ in $e^{il\varphi}$ makes no difference and we can restrict ourselves to the case $l \geq 0$. The mass spectrum and associated conformal dimensions of the fluctuations (4.4) are:

$$M_S(n,l) = \frac{2L}{R^2}\sqrt{(n+l)(n+l+1)} \;, \qquad \Delta_S = l+1 \;, \tag{4.5}$$

where $n \geq 0$, except for the case $l = 0$ where $n \geq 1$. Notice that for $n = l = 0$ the function $\xi_S(\varrho)$ is just constant. Moreover, M_S vanishes in this case and thus we can take the solution χ to be also independent of the Minkowski coordinates. This constant zero mode corresponds just to changing the value of the distance L and should not be considered as a true fluctuation. Therefore, we shall understand that $n \geq 0$ in (4.4) and (4.5), except in the case $l = 0$ where $n \geq 1$.

JHEP04(2006)037

4.1 Scalar fluctuations

Let us now study the fluctuations of the λ_i scalars. Notice that these fields are coupled through the Wess-Zumino term in (4.3). Actually, the equations of motion for the λ_i's derived from (4.3) are:

$$R^2\,\partial_a\Big[\rho\,(\rho^2+L^2)\,\mathcal{G}^{ab}\partial_b\lambda_i\Big]-4\rho\,(\rho^2+L^2)\,\epsilon^{ij}\partial_\varphi\lambda_j=0\;. \tag{4.6}$$

To solve these equations let us introduce the reduced variables $\bar{M}^2=-R^4L^{-2}k^2$, $\varrho=\rho/L$ and let us expand the λ_i's in modes as:

$$\lambda_i=e^{ikx}\,e^{-il\varphi}\,\xi_i(\rho)\;,\qquad i=1,2\;. \tag{4.7}$$

Then, the functions $\xi_i(\rho)$ satisfy the coupled equations:

$$\frac{1}{\varrho(1+\varrho^2)}\,\partial_\varrho\Big[\varrho\,(1+\varrho^2)^2\,\partial_\varrho\,\xi_i\Big]+\Bigg[\frac{\bar{M}^2}{1+\varrho^2}-l^2\left(1+\frac{1}{\varrho^2}\right)\Bigg]\,\xi_i\;+4il\,\epsilon^{ij}\xi_j=0\;. \tag{4.8}$$

In order to diagonalize this system of equations, let us define the following complex function:

$$w=\xi_1+i\,\xi_2\;, \tag{4.9}$$

which satisfies the differential equation:

$$\frac{1}{\varrho(1+\varrho^2)}\,\partial_\varrho\Big[\varrho\,(1+\varrho^2)^2\,\partial_\varrho\,w\Big]+\Bigg[\frac{\bar{M}^2}{1+\varrho^2}-l^2\left(1+\frac{1}{\varrho^2}\right)\Bigg]\,w\;+4l\,w=0\;. \tag{4.10}$$

Equation (4.10) can be solved in terms of a hypergeometric function, namely:

$$w^{(1)}=\varrho^{l}\,(1+\varrho^2)^{\,-\lambda-1}\;F(-\lambda+1,l-\lambda-1;l+1;-\varrho^2)\;, \tag{4.11}$$

where λ is related to $\bar{M}$ as in (2.41). Notice that $w^{(1)}$ is regular at $l=0$ for $l\geq 0$. Actually, since l can be negative in this case one can also consider the second solution of the hypergeometric equation, which is:

$$w^{(2)}=\varrho^{-l}\,(1+\varrho^2)^{\,-\lambda-1}\;F(-\lambda-1,-\lambda-l+1;1-l;-\varrho^2)\;. \tag{4.12}$$

By applying eq. (2.51) to the present case, we obtain that the conformal dimension of a fluctuation of the type (2.40) is just $\Delta=l-1$ or $\Delta=3-l$. Actually, it is straightforward to verify that the solutions of the differential equation (4.10) present two different behaviours at $\varrho\to\infty$, namely ϱ^{-l} and ϱ^{l-4}. The first behaviour corresponds to an operator with $\Delta=l-1$, while $\Delta=3-l$ is the dimension of an operator whose dual fluctuation behaves as ϱ^{l-4} for large ϱ. In the following we will refer to the fluctuations with $\Delta=l-1$ as W^l_+, while those with $\Delta=3-l$ will be denoted by W^l_-. These two branches [1] will be studied separately in their unitarity range $\Delta\geq 0$ by finding the truncations of the hypergeometric series of $w^{(1)}$ and $w^{(2)}$ with the appropriate behaviour at large ϱ.

[1]For $l=2$ both UV behaviours coincide and there is no distinction between the two branches. In this case there are solutions of the fluctuation equation which behave as $\varrho^{-2}\log\varrho$ for large ϱ.

JHEP04(2006)037

4.1.1 W_+^l fluctuations

Let us consider the solution $w^{(1)}$ in eq. (4.11) with the following truncation condition:

$$l-\lambda-1=-n\ ,\qquad n=0,1,2,\dots\ . \tag{4.13}$$

The resulting solution is:

$$w=\varrho^l\,(1+\varrho^2)^{\,-l-n}\ F(2-l-n,-n;l+1;-\varrho^2)\ . \tag{4.14}$$

One can check easily that $w\sim\varrho^{-l}$ for large ϱ if $l=1,n=0$ or $l\geq 2,n\geq 0$. Notice that $\Delta=l-1$ in this case and the unitarity range is $l\geq 1$. The mass spectrum becomes:

$$M_{W_+}(n,l)=\frac{2L}{R^2}\sqrt{(n+l-1)(n+l)}\ ,\qquad \Delta_{W_+}=l-1\ ,\qquad (l\geq 1)\ . \tag{4.15}$$

The $l=1$ fluctuation is a special case. Indeed, in this case one has $\Delta=0$ and, as n must vanish, M is zero, i.e. we have a massless mode despite of the fact that we have introduced a mass scale by separating the branes. As argued in ref. [24] this is related to the appearance of the Higgs branch on the field theory side. Let us look closer at this $l=1$, $n=0$ mode. In this case $M=\lambda=0$ and the hypergeometric function is just equal to one. Thus:

$$w\sim\frac{\rho}{\rho^2+L^2}\ , \tag{4.16}$$

where we have reintroduced the constant L. In particular, for large ρ

$$w\approx\frac{c}{\rho}\ ,\qquad(\rho\to\infty)\ , \tag{4.17}$$

where c is a constant. Let us consider the solution in which $k=0$ (which certainly has $M=0$). This solution does not depend on the coordinates (t,x^1). Let us introduce the φ dependence and define the following two complex variables:

$$\Lambda\equiv\lambda_1+i\lambda_2=x^2+ix^3\ ,\qquad Y=\rho\,e^{i\varphi}\ . \tag{4.18}$$

Then the modes we are studying satisfy for large ρ:

$$\Lambda Y\approx c\ ,\qquad(\rho\to\infty)\ , \tag{4.19}$$

which is just the holomorphic curve of the Higgs branch found in ref. [24] by looking at the vanishing of the F-terms of the susy theory.

It is worth to stress here the difference between the $l=1$ solution (4.16) and the fluctuations (4.14) for $l>1$. Indeed, in the latter case we get a full tower of solutions, depending on the excitation number n, whereas for $l=1$ we have only the single function (4.16). Moreover, the mass spectra (4.15) is simply related to the one corresponding to the transverse scalar fluctuations S^l only for $l>1$ (see section 4.3). One can regard (4.16) as a non-trivial solution in which the D3-brane probe is deformed at no cost along the directions of the worldvolume of the D3-brane of the background.

It is also interesting to point out that the differential equation (4.10) can be solved by taking $w=\rho^{-l}$ and $M=0$. This fact can be checked directly from eq. (4.10) or by taking

$\lambda \to 0$ in the solution (4.12). Notice, however that this solution is not well-behaved at $\rho \to 0$ for $l \geq 1$, contrary to what happens to the function written in eq. (4.16). A second solution with $M = 0$ can be obtained by putting $\lambda = 0$ in (4.11). This solution is regular at $\rho \to 0$. However, for $l > 1$ the hypergeometric function which results from taking $\lambda = 0$ in (4.11) contains logarithms for $L \neq 0$ and its interpretation in terms of a holomorphic curve is unclear to us.

4.1.2 W_-^l fluctuations

The allowed range of values of l for the fluctuations W_-^l is $l \leq 3$. We have found a discrete tower of states only for $l \leq 1$. As in the previous subsection, $l = 1$ is special. In this case the solutions regular at $\varrho = 0$ which decrease as ϱ^{-3} for large ϱ are:

$$w = \varrho\,(1+\varrho^2)^{\,-n-1}\; F(1-n, -n; 2; -\varrho^2)\;, \qquad (l=1)\;, \tag{4.20}$$

where $n \geq 1$. Indeed, this solution is just (4.14) for $l = 1$ and $n \geq 1$. Moreover, for $l \leq 0$ the solutions which behave as ϱ^{l-4} for large ϱ can be obtained by putting $-\lambda - l + 1 = -n$, with $n \geq 0$, on the solution $w^{(2)}$ of eq. (4.12). One gets:

$$w = \varrho^{-l}\,(1+\varrho^2)^{\,-2-n+l}\; F(-2-n+l, -n; 1-l; -\varrho^2)\;, \qquad (l \leq 0)\;. \tag{4.21}$$

The mass spectrum for $l \leq 1$ can be written as:

$$M_{W_-}(n,l) = \frac{2L}{R^2}\,\sqrt{(n+1-l)(n+2-l)}\;, \qquad \Delta_{W_-} = 3 - l\;, \qquad (l \leq 1)\;, \tag{4.22}$$

where it should be understood that $n \geq 1$ for $l = 1$ and $n \geq 0$ otherwise.

4.2 Vector fluctuations

We will now study the fluctuations of the worldvolume gauge field. We will try to imitate the discussion of section 3 for the D3-D5 system. Obviously, the analogue of the type I modes does not exists for a one-dimensional sphere. Let us analyze the spectra of the other two types of modes.

4.2.1 Type II modes

Let us consider the ansatz

$$A_\mu = \xi_\mu\,\phi(\rho)\,e^{ikx}\,e^{-il\varphi}\;, \qquad A_\rho = 0\;, \qquad A_\varphi = 0\;, \tag{4.23}$$

with ξ_μ being a constant vector such that $k^\mu \xi_\mu = 0$. In terms of the reduced variables ϱ and $\bar{M}$, the equation for $\phi(\rho)$ is:

$$\partial_\varrho\Big[\varrho\,\partial_\varrho\phi\Big] + \left[\frac{\varrho}{(\varrho^2+1)^2}\,\bar{M}^2 - \frac{l^2}{\varrho}\right]\phi = 0\;, \tag{4.24}$$

which is the same as for the transverse scalars χ. Therefore, the mass spectrum of these type II vector modes is just the same as in (4.5).

JHEP04(2006)037

4.2.2 Type III modes

We now adopt the ansatz:

$$A_\mu = 0\ , \qquad A_\rho = \phi(\rho)\, e^{ikx}\, e^{-il\varphi}\ , \qquad A_\varphi = \tilde{\phi}(\rho)\, e^{ikx}\, e^{-il\varphi}\ . \tag{4.25}$$

The equation for A_ρ is:

$$il\partial_\rho\tilde{\phi} - l^2\phi + M^2\, R^4\, \frac{\rho^2}{(\rho^2+L^2)^2}\,\phi = 0\ , \tag{4.26}$$

while the equation for A_φ yields:

$$\partial_\rho\left[\frac{(\rho^2+L^2)^2}{\rho}\,\Big(\partial_\rho\tilde{\phi} + il\phi\Big)\right] + \frac{M^2\, R^4}{\rho}\,\tilde{\phi} = 0\ . \tag{4.27}$$

Finally, the equation for A_μ gives a relation between ϕ and $\tilde{\phi}$, namely:

$$\rho\,\partial_\rho\Big(\rho\phi\Big) = il\tilde{\phi}\ . \tag{4.28}$$

For $l \neq 0$ we can use (4.28) to eliminate $\tilde{\phi}$ in favor of ϕ. The remaining equations reduce to the following equation for ϕ:

$$\partial_\varrho\Big[\varrho\,\partial_\varrho(\varrho\phi)\Big] + \left[\frac{\varrho^2}{(\varrho^2+1)^2}\,\bar{M}^2 - l^2\right]\phi = 0\ , \tag{4.29}$$

where we have already introduced the reduced variables ϱ and $\bar{M}$. The solution of (4.29) regular at $\rho = 0$ is:

$$\phi = \varrho^{l-1}\,(1+\varrho^2)^{-\lambda}\ F(-\lambda, l-\lambda; l+1; -\varrho^2)\ , \tag{4.30}$$

with λ being the quantity defined in (2.41). By imposing the quantization condition:

$$l - \lambda = -n\ , \qquad n = 0, 1, \ldots\ , \tag{4.31}$$

we get a tower of fluctuation modes which behaves as ρ^{-l-1} when $\rho \to \infty$. The corresponding mass levels and conformal dimensions are:

$$M_V(n,l) = \frac{2L}{R^2}\,\sqrt{(n+l)(n+l+1)}\ , \qquad \Delta_V = l+1\ , \tag{4.32}$$

which again coincide with the results obtained for the scalar modes.

4.3 Fluctuation/operator correspondence

The array corresponding to the D3-D3 intersection is:

$$\begin{array}{rccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ D3: & \times & \times & \times & - & - & - & - & - & - \\ D3': & \times & - & - & \times & \times & - & - & - & - \end{array} \tag{4.33}$$

where the $D3'$ is the probe brane. First of all, let us discuss the isometries of this configu-

JHEP04(2006)037

Field	Δ	SO(4)	$\mathrm{U}(1)_{23}$	$\mathrm{U}(1)_{45}$
q	1	$(0,0)$	0	1
$b, \tilde{b}$	0	$(0,0)$	$-1/2$	$1/2$
ψ^+	$1/2$	$(1/2,0)$	0	0
ψ^-	$1/2$	$(0,1/2)$	0	0

Table 3: Quantum numbers and dimensions of the fields of the D3-D3 intersection.

ration. Clearly, the addition of the brane probe breaks the SO(6) symmetry corresponding to rotations in the 456789 directions to $\mathrm{SO}(4) \times \mathrm{U}(1)_{45}$, where the $\mathrm{SO}(4) \approx \mathrm{SU}(2) \times \mathrm{SU}(2)$ factor is generated by rotations in the 6789 subspace and the $\mathrm{U}(1)_{45}$ corresponds to rotations in the plane spanned by coordinates 4 and 5. In addition we have an extra $\mathrm{U}(1)_{23}$ generated by the rotations in the 23 plane.

The field content of the defect theory can be obtained by reducing the $\mathcal{N} = 4$, $d = 4$ gauge multiplet down to two dimensions and by adding the corresponding $3-3'$ sector [24]. The resulting theory has $(4,4)$ supersymmetry in $d = 2$. In particular, two of the six $d = 4$ adjoint chiral scalar superfields give rise to a field Q whose lowest component (which we will denote by q) describes the fluctuations of the D3 in the directions 4 and 5. This field q is a singlet of SO(4) and $\mathrm{U}(1)_{23}$ and is charged under $\mathrm{U}(1)_{45}$. The strings stretched between the D3 and the D3' give rise to two chiral multiplets B and $\tilde{B}$ which are fundamental and antifundamental with respect to the gauge group. The lowest components of B and $\tilde{B}$ are two scalar fields b and $\tilde{b}$ which are singlets under SO(4) and are charged under $\mathrm{U}(1)_{23}$ and $\mathrm{U}(1)_{45}$. Moreover, the fermionic components of B and $\tilde{B}$ can be arranged in two SU(2) multiplets ψ^+ and ψ^- which are neutral with respect to the two U(1)'s and charged under one of the two SU(2)'s of the decomposition $\mathrm{SO}(4) \approx \mathrm{SU}(2) \times \mathrm{SU}(2)$. The dimensions and quantum numbers of the different fields just discussed are summarized in table 3.

In order to establish the fluctuation/operator dictionary in this case, let us determine the quantum numbers of the different fluctuations. The fluctuations in the directions 6789 (which are transverse to both types of D3-branes) will be denoted by S^l. Clearly they transform in the $(1/2,1/2)$ representation of SO(4) and are neutral under $\mathrm{U}(1)_{23}$. Moreover, since the rotations of the 45 plane are just those along the one-sphere of the probe worldvolume, the integer l is just the charge under $\mathrm{U}(1)_{45}$. The w coordinate parametrizes the 23 plane. Let us denote by W^l_+ and W^l_- to the two branches of w fluctuations. The $W^l_\pm$'s are SO(4) singlets and are charged under both $\mathrm{U}(1)_{23}$ and $\mathrm{U}(1)_{45}$. As in the D3-D5 case, the fluctuations of types II and III correspond, in the conformal limit, to the components of a vector field in AdS_3 and will be denoted by V^l. They are singlets under SO(4) and $\mathrm{U}(1)_{23}$. Table 4 is filled in with the dimensions and quantum numbers of the different fluctuations.

The mass spectrum $M_S(n,l)$ of the fluctuations S^l has been written in eq. (4.5). The masses of the other modes can be written in terms of $M_S(n,l)$ as:

$$M_{W_+}(n,l) = M_S(n,l-1) \ , \qquad (l \geq 2),$$

JHEP04(2006)037

Mode	Δ	SO(4)	$\mathrm{U}(1)_{23}$	$\mathrm{U}(1)_{45}$
S^l	$l+1$	$(1/2,1/2)$	0	$l\geq 0$
W^l_+	$l-1$	$(0,0)$	-1	$l\geq 1$
W^l_-	$3-l$	$(0,0)$	-1	$l\leq 1$
V^l	$l+1$	$(0,0)$	0	$l\geq 0$

Table 4: Quantum numbers and dimensions of the modes of the D3-D3 intersection.

$$
\begin{aligned}
&M_{W_-}(n,l)=M_S(n,1-l)\ , \qquad (l\leq 1),\\
&M_V(n,l)=M_S(n,l)\ , \qquad (l\geq 0).
\end{aligned}
\tag{4.34}
$$

Notice that the relation between M_S and M_{W_-} is consistent with the absence of the $n=0$ mode in the S^0 and W^1_- fluctuations.

In order to relate the different fluctuations to composite operators of the defect theory let us define following ref. [24] the operator $\mathcal{B}^l$ for $l\geq 1$ as:

$$
\mathcal{B}^l\equiv \tilde{b}q^{l-1}\,b\ . \tag{4.35}
$$

Notice that $\mathcal{B}^l$ has the same dimension and quantum numbers as the fluctuation W^l_+. Similarly, the operator $\mathcal{G}^l$ for $l\leq 1$, defined as:

$$
\mathcal{G}^l\equiv D_-\tilde{b}q^{\dagger 1-l}\,D_+b+D_+\tilde{b}q^{\dagger 1-l}\,D_-b\ , \tag{4.36}
$$

where $D_\pm=D_0\pm D_1$, has the right properties to be identified with the dual of the fluctuations W^l_-. Moreover, to define the operator dual to S^l we have to build a vector of SO(4). The natural objects to build an operator of this sort are the spinor fields $\psi^\pm$. Indeed, such an operator can be written as:

$$
\mathcal{C}^{\mu l}\equiv \sigma^\mu_{ij}\,(\epsilon_{ik}\bar{\psi}^+_k\,q^l\psi^-_j+\epsilon_{jk}\bar{\psi}^-_k\,q^l\psi^-_i\,)\ , \tag{4.37}
$$

where $l\geq 0$ and μ is a SO(4) index. Therefore, according to the proposal of ref. [24], we have

$$
S^l\sim\mathcal{C}^{\mu l}\ , \qquad W^l_+\sim\mathcal{B}^l\ , \qquad W^l_-\sim\mathcal{G}^l\ . \tag{4.38}
$$

As argued in ref. [24], the fluctuations W^l_+ for $l=1$ are dual to the field $\mathcal{B}^1=\tilde{b}b$, which parametrizes the classical Higgs branch of the theory, whereas for higher values of l these fluctuations correspond to other holomorphic curves. Moreover, the field $\mathcal{C}^{\mu l}$ is a BPS primary and $\mathcal{G}^{1-l}$ is a two supercharge descendant of this primary. Notice that this is consistent with our relation (4.34) between the masses of the S^l and W^l_- fluctuations. Lastly, the fluctuations V^l are dual to two-dimensional vector currents. The dual operator at the bottom of the Kaluza-Klein tower is a global U(1) current $\mathcal{J}^M_B$:

$$
V^0\sim\mathcal{J}^M_B\ . \tag{4.39}
$$

JHEP04(2006)037

The expression of $\mathcal{J}_B^M$ has been given in ref. [24], namely:

$$\mathcal{J}_B^M \equiv \psi_i^\alpha \, \rho^M_{\alpha\beta} \, \psi_i^\beta + i\bar{b} \overleftrightarrow{D}^M b + i\tilde{b} \, \overleftrightarrow{D}^M \bar{\tilde{b}} \, , \qquad (M = 0, 1) \, , \tag{4.40}$$

where $\alpha, \beta = +, -$ and ρ^M are Dirac matrices in two dimensions.

5. Concluding remarks

In this paper we have studied the fluctuation spectra of brane probes in the near-horizon background created by a stack of other branes. In the context of the generalization of the gauge/gravity correspondence proposed in refs. [4, 5] these fluctuations are dual to open strings stretching between the two types of branes of the intersection and can be identified, on the field theory side of the correspondence, with composite operators made up from hypermultiplets in the fundamental representation of the gauge group. We have mainly studied the cases of D5- and D3-brane probes moving in the $AdS_5 \times S^5$ geometry. In these two cases, if the brane reaches the origin of the holographic coordinate, it wraps an $AdS_{d+2} \times S^d$ $(d = 2, 1)$ submanifold of the $AdS_5 \times S^5$ background and the corresponding field theory duals are defect conformal field theories with a fundamental hypermultiplet localized at the defect. The spectra of conformal dimensions and the precise mapping between probe fluctuations and operators of the dual defect theory for the D3-D5 and D3-D3 intersections were obtained in refs. [21] and [24] respectively.

By allowing a finite separation between the probe and the origin of the holographic direction we introduce an explicit mass scale in the problem, which is related to the mass of the hypermultiplet. The system is no longer conformal and develops a mass gap. By studying the fluctuations of the probes we have obtained the mass spectrum of the corresponding open string degrees of freedom. Remarkably, for the D3-D5 and D3-D3 systems the full set of differential equations for the fluctuations can be solved in terms of the hypergeometric function and the corresponding mass spectra can be obtained analytically. These mass spectra display some degeneracies which are consistent with the structure of the supermultiplets found in refs. [21, 24] for the corresponding dual operators.

In the appendices we have also studied the case of supersymmetric D-brane probes in the background of a Dp-brane for $p \neq 3$. In these cases the fluctuation equations can also be decoupled although one needs to use numerical methods or WKB estimates in order to get the mass spectra. The fluctuation/operator dictionary for these intersections has not been worked out in detail in the literature. However, we have obtained relations between the masses of the different modes which closely resemble the ones found for the $AdS_5 \times S^5$ background. Moreover, one has to keep in mind [1] that, in order to trust these supergravity solutions, both the curvature in string units and the dilaton must be small. This fact introduces restrictions in the range of the holographic coordinate for which the correspondence between the supergravity and gauge theory descriptions is valid [32]. It is interesting to point out that in some cases we can avoid that our fluctuations enter the "bad" region by selecting appropriately the value of the distance L.

This paper is a contribution to the program which aims at the extension of the gauge/gravity correspondence to the case in which the field theory dual contains mat-

JHEP04(2006)037

ter in the fundamental representation. This program is still in progress and at the present time it has some unsolved problems, which are also reflected in our results. Let us comment on some of them. First of all, notice that all mass spectra we have found grow linearly with the excitation number n for large n. This behaviour is common to all the cases in which the masses are extracted from the analysis of the fluctuations of D-branes, both in non-confining and confining backgrounds. This same type of spectrum is obtained in the effective holography models [33]. It has been argued in ref. [34] that the spectrum of highly excited mesons in confining gauge theories should be a linear function of $\sqrt{n}$ for large n. Even though the arguments in favor of the $M \sim \sqrt{n}$ behaviour do not apply to the systems studied here, this discrepancy could lead us to think that the approach based on the small fluctuations of brane probes needs to be corrected. However, the present status of this question is unclear and this should be considered as an open problem. Notice that, for large spin, the open string can be treated semiclassically. For the D3-D7 intersection this analysis was performed in ref. [7], while open spinning strings for the defect conformal field theory were studied in refs. [35] in connection with their relation with integrable spin chains [36, 37].

Another criticism that one could make to the approach followed here is the fact that we have employed the probe approximation and we have neglected the backreaction of the branes on the geometry. Indeed, in some cases, one can construct a supergravity background representing the localized intersection [38]. However, these backgrounds are rather complicated and it is not easy to extract information about the gauge theory dual. Maybe, in order to go beyond the probe approximation, it would be more fruitful to follow the approach recently proposed in [39] for the $\mathcal{N} = 1$ theories, where the supergravity action is supplemented with the action of the brane, which has been conveniently smeared, and a solution of the equations of motion for the supergravity plus brane system is found. In the approach of [39] the adjoint (color) degrees of freedom are represented by fluxes, whereas the fundamental (flavor) fields are generated by branes.

Despite the limitations of our approach just discussed we think that it provides a non-trivial realization of the holographic idea (see [40] for a rigorous treatment) and it would be interesting in the future to look at some generalizations of our results. For example, it has been argued in [41] that the Higgs branch of the D3-D7 system can be generated by turning on a non-zero instanton field on the D7-brane probe. This result could be clearly generalized for any Dp-D(p+4) intersection. Similarly, in the Dp-D(p+2) system one can switch a non-zero flux of the worldvolume gauge field across the S^2. As checked in ref. [23], in order to preserve some fraction of supersymmetry one has to introduce some bending on the defect boundary. Clearly, the analysis of the fluctuations around such configurations could shed light to understand the nature of the deformation induced by the flux on the field theory side.

It is also of great interest to look at defect theories with reduced supersymmetry. One of such theories is obtained by embedding a D5-brane in the $AdS_5 \times T^{1,1}$ geometry. The precise form of the embedding in this case can be found in [17]. Actually, the $T^{1,1}$ space can be substituted by any Sasaki-Einstein space [42], as illustrated in [43] for the $Y^{p,q}$ manifold. The supersymmetric defects in the Maldacena-Nuñez background have been obtained in ref. [44].

We hope to make some progress along these lines in the future.

JHEP04(2006)037

Acknowledgments

We are grateful to J.D. Edelstein, J. Erdmenger, J. Mas, P. Merlatti, C. Núñez, A. Paredes and M. Schvellinger for discussions and useful comments. This work was supported in part by MCyT, FEDER and Xunta de Galicia under grant FPA2005-00188 and by the EC Commission under grants HPRN-CT-2002-00325 and MRTN-CT-2004-005104.

A. Change of variables for the exact spectra

Let us consider an equation of the type:

$$z(1-z)\,\phi'' + (\alpha+\beta z)\,\phi' + [\gamma + \delta\, z^{-1} + \epsilon\,(1-z)^{-1}]\,\phi = 0\ , \tag{A.1}$$

where $\phi = \phi(z)$, the prime denotes derivative with respect to z and $\alpha, \beta, \gamma, \delta$ and ϵ are constants. The solution of this equation can be written as

$$\phi(z) = z^f\,(1-z)^g\,P(z)\ , \tag{A.2}$$

where $P(z)$ satisfies the hypergeometric equation

$$z(1-z)\,P'' + [c-(a+b+1)z]\,P' - ab\,P = 0\ . \tag{A.3}$$

The values of the exponents f and g are

$$\begin{aligned} f &= \frac{1-\alpha+\lambda_1\sqrt{(\alpha-1)^2-4\delta}}{2}\ ,\\ g &= \frac{\alpha+\beta+1+\lambda_2\sqrt{(\alpha+\beta+1)^2-4\epsilon}}{2}\ , \end{aligned} \tag{A.4}$$

where λ_1 and λ_2 are signs which can be chosen by convenience. Moreover, a, b and c are given by:

$$\begin{aligned} a &= f+g-\frac{1+\beta+\sqrt{4\gamma+(\beta+1)^2}}{2}\ ,\\ b &= f+g-\frac{1+\beta-\sqrt{4\gamma+(\beta+1)^2}}{2}\ ,\\ c &= \alpha+2f\ . \end{aligned} \tag{A.5}$$

There are two solutions for $P(z)$ in terms of the hypergeometric function. The first one is:

$$P(z) = F(a,b;c;z)\ . \tag{A.6}$$

The second solution is:

$$P(z) = z^{1-c}\,F(a-c+1,b-c+1;2-c;z)\ . \tag{A.7}$$

JHEP04(2006)037

B. Fluctuations of the Dp-D(p+2) system

In this section we analyze the full set of fluctuations of the $(p-1|Dp \perp D(p+2))$ intersections for $p<5$. Many of the results are direct generalizations of those corresponding to the D3-D5 system studied in section 3. Our starting point is the Dirac-Born-Infeld lagrangian density of the D(p+2)-brane probe, which is the sum of the Born-Infeld part $\mathcal{L}_{BI}$ and the Wess-Zumino part $\mathcal{L}_{WZ}$, which for the case at hand are given by:

$$\mathcal{L}_{BI} = -e^{-\phi}\sqrt{-\det(g+F)}\,, \qquad \mathcal{L}_{WZ} = P[C^{(p+1)}]\wedge F\,, \tag{B.1}$$

where, again, we are taking the D(p+2)-brane tension equal to one and $F=dA$ is the worldvolume gauge field. We shall expand the action around the configuration in which both types of branes are separated a fixed distance L. Let χ be the scalars transverse to both types of branes and let X denote the scalar which is transverse to the probe and that is directed along the worldvolume of the Dp-branes of the background (i.e. $X\equiv x^p$). At quadratic order, the Born-Infeld lagrangian becomes:

$$\mathcal{L}_{BI} = -\rho^2\sqrt{\tilde g}\left[\frac{1}{2}\left[\frac{R^2}{\rho^2+L^2}\right]^{\frac{7-p}{4}}\mathcal{G}^{ab}\,\partial_a\chi\partial_b\chi+\frac{1}{2}\left[\frac{\rho^2+L^2}{R^2}\right]^{\frac{7-p}{4}}\mathcal{G}^{ab}\partial_a X\partial_b X+\frac{1}{4}F_{ab}F^{ab}\right]\,, \tag{B.2}$$

where $\mathcal{G}_{ab}$ is the induced metric of the unperturbed static configuration, given by:

$$\mathcal{G}_{ab}d\xi^a d\xi^b = \left[\frac{\rho^2+L^2}{R^2}\right]^{\frac{7-p}{4}} dx^2_{1,p-1}+\left[\frac{R^2}{\rho^2+L^2}\right]^{\frac{7-p}{4}}\left[d\rho^2+\rho^2\,d\Omega_2^2\right]\,. \tag{B.3}$$

Let us now write the explicit form of the Wess-Zumino term $\mathcal{L}_{WZ}$. Recall that the Ramond-Ramond (p+1)-form potential $C^{(p+1)}$ has components along the Minkowski directions $x^0\cdots x^p$ (see eq. (2.15)). Therefore, it is clear from the expression of $\mathcal{L}_{WZ}$ in (B.1) that the potential $C^{(p+1)}$ induces a coupling between the scalar X and the components of the worldvolume field F along the radial coordinate ρ and along the angular directions of the two-sphere (which we denote by θ and φ). Actually, one can prove that:

$$\mathcal{L}_{WZ} = \left[\frac{\rho^2+L^2}{R^2}\right]^{\frac{7-p}{2}}\left[\partial_\rho X\,F_{\theta\varphi}+\partial_\theta X\,F_{\varphi\rho}+\partial_\varphi X\,F_{\rho\theta}\right]\,. \tag{B.4}$$

Integrating by parts, one can rewrite $\mathcal{L}_{WZ}$ as:

$$\mathcal{L}_{WZ} = -\frac{7-p}{2}\,\frac{\rho}{R^{7-p}}\left[\rho^2+L^2\right]^{\frac{5-p}{2}} X\epsilon^{ij}F_{ij}\,, \tag{B.5}$$

where i,j denote components along the S^2 and $\epsilon^{ij}=\pm 1$. As a check of the expressions (B.2) and (B.5) for $\mathcal{L}_{BI}$ and $\mathcal{L}_{WZ}$ notice that, by taking $p=3$, they reduce to the ones found in section 3 for the D3-D5 system.

Let us study first fluctuations of the scalars χ which, as is evident from eqs. (B.2) and (B.5), are decoupled from the other modes. The corresponding equations of motion

JHEP04(2006)037

$(0\|D1 \perp D3)$ with $l=0$		
n	WKB	Numerical
0	7.40	5.68
1	34.54	32.40
2	81.42	79.06
3	148.04	145.52
4	234.40	231.76
5	340.50	337.76

$(1\|D2 \perp D4)$ with $l=0$		
n	WKB	Numerical
0	5.73	4.34
1	25.21	23.66
2	58.44	56.80
3	105.42	103.74
4	166.15	164.44
5	240.63	238.92

$(3\|D4 \perp D6)$ with $l=0$		
n	WKB	Numerical
0	2.15	1.68
1	7.18	6.78
2	15.07	14.72
3	25.84	25.58
4	39.48	39.34
5	55.99	56.02

Table 5: Numerical and WKB values of M^2 (in units of $R^{7-p}L^{p-5}$) for the S^l modes of the Dp-D(p+2) intersection for $p = 1, 2, 4$ and $l = 0$.

can be obtained from the ones of a general intersection with $p_2 = p + 2$ and $d = p - 1$ and with the γ_i written in eq. (2.14). The resulting WKB spectra is obtained by plugging these parameters in the general expression (2.65). One gets:

$$M_S^{WKB}(n,l) = 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\left(\frac{7-p}{4}\right)}{\Gamma\left(\frac{5-p}{4}\right)}\,\sqrt{(n+1)\left(n+\frac{7-p}{5-p}\left(l+\frac{1}{2}\right)\right)}\,, \qquad \text{(B.6)}$$

where R is given by eq. (2.13). Moreover, the behavior of the fluctuation ξ when $\varrho \to 0$ is of the form $\xi \sim \varrho^{\gamma}$, where γ satisfies a quadratic equation whose two roots are $\gamma = l, -l-1$. Clearly, the regular solution should correspond to the root $\gamma = l$, i.e. ξ must behave as:

$$\xi \sim \varrho^{l}\,, \qquad\qquad \text{as} \qquad \varrho \to 0\,. \qquad \text{(B.7)}$$

In table 5 we compare the numerical results for $\bar{M}$ with the WKB formulas for some $(p-1|Dp \perp D(p+2))$ intersections.

We now want to address the analysis of the remaining fluctuation modes. First of all, from the expressions of $\mathcal{L}_{BI}$ and $\mathcal{L}_{WZ}$ (eqs. (B.2) and (B.5)) it is straightforward to find the equation of motion of the scalar X, namely:

$$R^{\frac{7-p}{2}}\partial_a\left(\rho^2\sqrt{\tilde{g}}\left[\rho^2+L^2\right]^{\frac{7-p}{4}}\mathcal{G}^{ab}\partial_b X\right) - \frac{7-p}{2}\,\rho\,\left[\rho^2+L^2\right]^{\frac{5-p}{2}}\epsilon^{ij}F_{ij} = 0\,, \qquad \text{(B.8)}$$

while those of the gauge field are:

$$R^{7-p}\,\partial_a\left[\rho^2\sqrt{\tilde{g}}\,F^{ab}\right] - (7-p)\,\rho\,\left[\rho^2+L^2\right]^{\frac{5-p}{2}}\epsilon^{bi}\partial_i X = 0\,, \qquad \text{(B.9)}$$

where ϵ^{bi} is zero unless b is an index along the two-sphere. As in the D3-D5 case, the scalar X is coupled to the gauge field strength F_{ij} along the two-sphere. To decouple these two fields we will follow the same strategy as in the exactly solvable case of section 3. First of all, we introduce the two types of vector spherical harmonics on S^2, namely Y_i^l and $\hat{Y}_i^l$. These functions were defined in eqs. (3.7) and (3.9) respectively. Subsequently, we will define the three types of modes, namely I, II and III, exactly as in the D3-D5 case

JHEP04(2006)037

and we will be able to decouple the corresponding equations of motion. For the general $(p-1|Dp \perp D(p+2))$ intersection these equations cannot be solved analytically, although they are easy to solve numerically and WKB expressions for the mass levels can be readily found.

B.1 Type I modes

Let us adopt an ansatz for the gauge field and the scalar X as in eqs. (3.11) and (3.12). The equation of motion of X (eq. (B.8)) becomes:

$$\rho^2 R^{7-p}\partial_\mu\partial^\mu\Lambda + \partial_\rho\left[\rho^2\left[\rho^2+L^2\right]^{\frac{7-p}{2}}\partial_\rho\Lambda\right] - \\ - l(l+1)\left[\rho^2+L^2\right]^{\frac{7-p}{2}}\Lambda - (7-p)l(l+1)\rho\left[\rho^2+L^2\right]^{\frac{5-p}{2}}\phi = 0\ , \tag{B.10}$$

while that of the gauge field (eq. (B.9)) reduces to:

$$R^{7-p}\partial_\mu\partial^\mu\phi + \partial_\rho\left[\left[\rho^2+L^2\right]^{\frac{7-p}{2}}\partial_\rho\phi\right] - \\ -\frac{l(l+1)}{\rho^2}\left[\rho^2+L^2\right]^{\frac{7-p}{2}}\phi - (7-p)\rho\left[\rho^2+L^2\right]^{\frac{5-p}{2}}\Lambda = 0\ . \tag{B.11}$$

Let us next define $V=\rho\Lambda$ as in the D3-D5 case and the following differential operator:

$$\Delta^2_{(p+1)}\,\Psi \equiv \frac{1}{(\rho^2+L^2)^{\frac{5-p}{2}}}\left[R^{7-p}\,\partial_\mu\partial^\mu\,\Psi + \partial_\rho\Big[\,(\rho^2+L^2)^{\frac{7-p}{2}}\,\partial_\rho\,\Psi\Big]\right]\ . \tag{B.12}$$

With these definitions the equations for the fluctuations become the following system of coupled differential equations:

$$\begin{aligned}\Delta^2_{(p+1)}\,V &= \left[l(l+1)+7-p+l(l+1)\,\frac{L^2}{\rho^2}\right]V + (7-p)l(l+1)\,\phi\ ,\\ \Delta^2_{(p+1)}\,\phi &= \left[l(l+1)+l(l+1)\,\frac{L^2}{\rho^2}\right]\phi + (7-p)\,V\ .\end{aligned} \tag{B.13}$$

Let us decouple the equations (B.13) by following a procedure similar to the one employed in [21] for the $L=0$ case. With this purpose we consider first the system (B.13) for $\rho\to\infty$. In this UV limit (B.13) reduces to:

$$\Delta^2_{(p+1)}\begin{pmatrix}V\\ \phi\end{pmatrix} = \mathcal{M}\begin{pmatrix}V\\ \phi\end{pmatrix}\ , \tag{B.14}$$

with $\mathcal{M}$ being the following constant matrix:

$$\mathcal{M} = \begin{pmatrix} l(l+1)+7-p & (7-p)l(l+1)\\ 7-p & l(l+1)\end{pmatrix}\ . \tag{B.15}$$

JHEP04(2006)037

The UV system (B.14) is readily decoupled by finding the linear combinations of the functions V and ϕ that diagonalize the matrix $\mathcal{M}$. Interestingly, the eigenvalues of $\mathcal{M}$ are integers and given by:

$$(l+1)(l+7-p)\ , \qquad l(l+p-6)\ , \tag{B.16}$$

while the corresponding decoupled functions are:

$$\begin{aligned} Z^{+} &= V + l\phi\ , \\ Z^{-} &= V - (l+1)\,\phi\ . \end{aligned} \tag{B.17}$$

Let us now write the equations for $Z^{\pm}$ for finite ρ. Remarkably, by substituting the definitions of $Z^{\pm}$ in (B.13), one can verify that these equations are still decoupled for finite ρ and take the form:

$$\begin{aligned} \Delta^2_{(p+1)}\, Z^{+} &= (l+1)\Big(l+7-p+l\frac{L^2}{\rho^2}\Big)\, Z^{+}\ , \\ \Delta^2_{(p+1)}\, Z^{-} &= l\Big(l+p-6+(l+1)\frac{L^2}{\rho^2}\Big)\, Z^{-}\ . \end{aligned} \tag{B.18}$$

As a check, let us notice that when $\rho \to \infty$ the right-hand sides of the two equations in (B.18) are just the eigenvalues of $\mathcal{M}$, displayed in (B.16), multiplying the functions $Z^{\pm}$. To analyze these equations let us proceed as in the D3-D5 case and separate variables as:

$$Z^{\pm} = e^{ikx}\,\xi^{\pm}(\rho)\ . \tag{B.19}$$

Except for the $p=3$ case, the resulting decoupled ordinary differential equations for $\xi^{+}(\rho)$ and $\xi^{-}(\rho)$ cannot be solved in an analytic form. However, we can extract some important qualitative information on the behaviour of their solutions by rewriting them as Schrödinger equations. In order to perform this analysis, let us introduce the reduced quantities ϱ and $\bar{M}$ as:

$$\varrho = \frac{\rho}{L}\ , \qquad \bar{M}^2 = -R^{7-p}\,L^{p-5}\,k^2\ . \tag{B.20}$$

Moreover, by changing variables as

$$e^{y} = \varrho\ , \qquad \psi^{\pm} = \frac{\Big[1+\varrho^2\Big]^{\frac{7-p}{4}}}{\sqrt{\varrho}}\ \xi^{\pm}\ , \tag{B.21}$$

we can convert the fluctuation equations (B.18) into the Schrödinger equation

$$\partial_y^2\,\psi^{\pm} - V_{\pm}(y)\,\psi^{\pm} = 0\ , \tag{B.22}$$

where the potentials $V_{\pm}(y)$ are given by:

$$\begin{aligned} V_{\pm}(y) = &\Big(l+\frac{1}{2}\Big)^2 \pm (7-p)\Big(l+\frac{1}{2}\pm 1\Big)\frac{e^{2y}}{1+e^{2y}} + \\ &+\frac{1}{4}\,(7-p)(3-p)\,\frac{e^{4y}}{(1+e^{2y})^2} - \bar{M}^2\frac{e^{2y}}{(1+e^{2y})^{\frac{7-p}{2}}}\ . \end{aligned} \tag{B.23}$$

JHEP04(2006)037

By looking at the asymptotic values of the potentials $V_\pm$ at $y \to \pm\infty$ we can get the behaviour of the fluctuations $\xi^\pm$ at $\varrho \approx 0, \infty$. Indeed, from the potentials (B.23) we obtain:

$$\lim_{y\to-\infty} V_\pm(y) = \Big(l+\frac{1}{2}\Big)^2\,, \qquad \lim_{y\to+\infty} V_\pm(y) = \Big(l+\frac{1}{2}\pm\frac{7-p}{2}\Big)^2\,. \tag{B.24}$$

From these values one can prove that for $\varrho \approx 0$ the functions $\xi^\pm$ behave as:

$$\xi^\pm \approx c_1\,\varrho^{l+1} + c_2\,\varrho^{-l}\,, \qquad (\rho\approx 0)\,, \tag{B.25}$$

while for $\varrho \to \infty$ one gets:

$$\begin{aligned} \xi^+ &\approx d_1^+\,\varrho^{-(l+7-p)} + d_2^+\,\varrho^{l+1}\,, \\ \xi^- &\approx d_1^-\,\varrho^{-l} + d_2^-\,\varrho^{l+p-6}\,, \qquad (\varrho\to\infty)\,. \end{aligned} \tag{B.26}$$

Obviously, the regular solutions at $\varrho \to 0$ for any $l \geq 0$ are those which behave as ϱ^{l+1} for small ϱ, i.e. those with $c_2 = 0$ in eq. (B.25). Moreover, we should also impose that $\xi^\pm$ vanish when $\varrho \to \infty$. From eq. (B.26) we notice that the behaviour of the ξ^+ modes is different from that of the ξ^-'s and, therefore, both cases must be analyzed separately. Clearly, only those ξ^+ modes for which $d_2^+ = 0$ are normalizable. In analogy with the D3-D5 case studied in section 3, let us call I_+^l to these modes. On the contrary, the two exponents of ϱ in the expression giving the $\varrho \to \infty$ behaviour of ξ^- in (B.26) could be negative and, thus, we have two types of modes. Generalizing the case of the D3-D5 system, the modes with $d_2^- = 0$ ($d_1^- = 0$) will be denoted by I_-^l ($\tilde I_-^l$). Therefore, the different behaviours at $\varrho \to \infty$ are:

$$\begin{aligned} I_+^l &\quad\Longrightarrow\quad \xi^+ \sim \varrho^{-(l+7-p)}\,, \qquad (l\geq 0)\,, \\ I_-^l &\quad\Longrightarrow\quad \xi^- \sim \varrho^{-l}\,, \qquad (l\geq 1)\,, \\ \tilde I_-^l &\quad\Longrightarrow\quad \xi^- \sim \varrho^{l+p-6}\,, \qquad (1\leq l< 6-p)\,, \end{aligned} \tag{B.27}$$

where, we have taken into account that for $l = 0$ only the ξ^+ modes exist (see section 3). As in the D3-D5 intersection, notice that the $\tilde I_-^l$ modes only exist for a finite number of values of the angular quantum number l.

The behaviours displayed in (B.27) can be used to characterize the different types of modes in the numerical calculations of the energy levels (see below). It is also interesting to study these levels in the WKB approximation. In particular it is interesting to find out how the WKB approximation distinguishes between the two types of ξ^- modes. The key observation in this respect is to realize that within the WKB approach the solution of the wave equation that is selected is that in which the "wave function" $\psi^\pm$ vanishes when we move beyond the turning points of the potential. Using the fact that $\psi^\pm \approx \varrho^{3-\frac{p}{2}}\,\xi^\pm$ for large ϱ, one can check immediately that for the I_+^l modes $\psi^+ \approx \varrho^{-4-l+\frac{p}{2}}$, which always vanish for $\varrho \to \infty$. On the contrary, for the I_-^l ($\tilde I_-^l$) modes $\psi^- \approx \varrho^{3-l-\frac{p}{2}}$ ($\psi^- \approx \varrho^{-3+l+\frac{p}{2}}$), which means that for $l \geq 3-\frac{p}{2}$ the WKB approximation picks up the I_-^l branch whereas

JHEP04(2006)037

$(0\|D1 \perp D3)$		
n	$I_+^{l=0}$	$\tilde{I}_-^{l=1}$
0	27.06	22.04
1	69.41	63.75
2	131.38	125.26
3	213.01	206.55
4	314.36	307.60
5	435.41	428.41

$(1\|D2 \perp D4)$		
n	$I_+^{l=0}$	$\tilde{I}_-^{l=1}$
0	21.04	15.00
1	52.13	43.63
2	96.92	86.02
3	155.43	142.17
4	227.68	212.06
5	313.67	259.71

Table 6: $\bar{M}^2$ for the $I_+^{l=0}$ and $\tilde{I}_-^{l=1}$ modes of the D1-D3 intersection (left) and D2-D4 system (right).

for $l \leq 3 - \frac{p}{2}$ the $\tilde{I}_-^l$ modes are selected[2]. The sign of $l - 3 + \frac{p}{2}$ is relevant when one computes the quantity β_2, defined in (2.60), which in turn is needed to apply the WKB energy formula (2.61). Applying these ideas to the case at hand, the WKB energy levels for the $I_\pm$ modes are given by:

$$M_{I_\pm}^{WKB}(n,l) = 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\left(\frac{7-p}{4}\right)}{\Gamma\left(\frac{5-p}{4}\right)}\,\sqrt{(n+1)\left(n+\frac{7-p}{5-p}\left(l+\frac{1}{2}\pm 1\right)\right)}\,, \tag{B.28}$$

whereas the WKB energy levels for the $\tilde{I}_-$ modes are:

$$M_{\tilde{I}_-}^{WKB}(n,l) = 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\left(\frac{7-p}{4}\right)}{\Gamma\left(\frac{5-p}{4}\right)}\,\sqrt{(n+1)\left(n+\frac{7-p}{5-p}+\frac{3-p}{5-p}\left(l+\frac{1}{2}\right)\right)}\,. \tag{B.29}$$

Let us use the WKB mass formulae (B.28) and (B.29) to notice some regularities, which can be subsequently checked with the numerical calculations. First of all, by direct comparison of (B.6) and (B.28) one gets that the masses of the $I_\pm$ modes are related to those of the scalar fluctuations as:

$$M_{I_\pm}^{WKB}(n,l) = M_S^{WKB}(n,l\pm 1)\,. \tag{B.30}$$

We have also verified that this relation holds numerically. In the case of the D3-D5 intersection the analogue of (B.30), namely (3.51), was crucial to organize the different fluctuations in massive supermultiplets. Notice also that the WKB formula (B.29) for the masses of the $\tilde{I}_-$ modes gives a result which is independent of l only for $p = 3$ and, actually, it coincides with the exact result (3.33) in the case of the D3-D5 system. In table 6 we list the mass levels obtained numerically for the I_+ fluctuations for $l = 0$ and for the $\tilde{I}_-$ modes for $l = 1$. As mentioned above the levels for $I_-^{l=1}$ are equal, within the accuracy of our numerical calculation, to the ones of the $S^{l=0}$ fluctuations, which were given at the beginning of this section.

[2] Notice that for $l = 3 - \frac{p}{2}$ the two types of modes behave in the same way for $\varrho \to \infty$ and there is no real distinction between them. This case can only happen for even p.

JHEP04(2006)037

B.2 Type II modes

As a generalization of the case of the D3-D5 intersection, let us consider a configuration in which the scalar field X vanishes and the vector field has only non-vanishing components along the Minkowski directions x^μ, which are given by the ansatz of eqs. (3.34) and (3.37). It is immediate to verify that the equations for the gauge field components A_ρ and A_i are identically satisfied, whereas the equation of motion for A_μ, in terms of the reduced variables $\varrho = \rho/L$ and $\bar{M}^2 = R^{7-p}\, L^{p-5}\, M^2$, is equivalent to:

$$\partial_\varrho(\varrho^2\,\partial_\varrho\chi\,) + \left[\bar{M}^2\;\frac{\varrho^2}{(\varrho^2+1\,)^{\frac{7-p}{2}}} - l(l+1)\right]\chi = 0\;. \tag{B.31}$$

Eq. (B.31) is exactly the same as the one corresponding to the scalar fluctuations. Therefore we conclude that, as it happened for the D3-D5 system, the spectrum of type II vector modes is identical to the one corresponding to the scalar modes.

B.3 Type III modes

Let us consider the type III modes with the same ansatz (3.39) as in the D3-D5 system. The equation for A_ρ is now:

$$\rho^2\, R^{7-p}\partial_\mu\partial^\mu\phi - l(l+1)\Big[\rho^2+L^2\Big]^{\frac{7-p}{2}}(\phi - \partial_\rho\tilde{\phi}) = 0\;. \tag{B.32}$$

The equation for A_i is:

$$R^{7-p}\partial_\mu\partial^\mu\tilde{\phi} + \partial_\rho\Big[(\rho^2+L^2)^{\frac{7-p}{2}}\,(\partial_\rho\tilde{\phi}-\phi)\Big] = 0\;, \tag{B.33}$$

and the equation for A_μ is exactly the same as in the D3-D5 intersection, namely (3.43). Using these relations, the two equations written above are equivalent. Separating variables in ϕ as in the D3-D5 case, we obtain the following equation:

$$\frac{1}{\varrho^2}\,\partial^2_\varrho\Big(\varrho^2\zeta\Big) + \left[\frac{\bar{M}^2}{(1+\varrho^2)^{\frac{7-p}{2}}} - \frac{l(l+1)}{\varrho^2}\right]\zeta = 0\;. \tag{B.34}$$

In order to obtain the spectrum derived from this equation, let us perform a change of variables to convert (B.34) into the Schrödinger equation (2.34). This change of variables is the following:

$$e^y = \varrho\;, \qquad \psi = \varrho^{\frac{3}{2}}\;\zeta\;. \tag{B.35}$$

The potential V of (2.34) after this change of variables is:

$$V = \Big(l+\frac{1}{2}\Big)^2 - \bar{M}^2\;\frac{e^{2y}}{(1+e^{2y})^{\frac{7-p}{2}}}\;, \tag{B.36}$$

which is just the same as the one corresponding to the scalar excitations. Therefore, we conclude that the mass levels of these modes of type III are the same as the ones corresponding to the scalar fluctuations. Putting together the results corresponding to the modes of types II and III, we conclude that the vector fluctuations have a mass spectrum which coincides with that of the scalar modes, namely:

$$M_V(n,l) = M_S(n,l)\;, \tag{B.37}$$

a result which generalizes the one for the D3-D5 intersection.

JHEP04(2006)037

C. Fluctuations of the Dp-Dp system

The lagrangian density of a Dp-brane probe in the background generated by a stack of Dp-branes is the sum of the Born-Infeld and Wess-Zumino terms, which are given by:

$$\mathcal{L}_{BI} = -e^{-\phi}\sqrt{-\det(g+F)}\;, \qquad \mathcal{L}_{WZ} = P[C^{(p+1)}]\;. \tag{C.1}$$

In this section we will analyze the $(p-2|Dp \perp Dp)$ intersection for $2 \le p < 5$. The induced metric on the worldvolume of the probe for a static configuration of such intersection in which the branes are separated a constant distance L is given by:

$$\mathcal{G}_{ab}d\xi^a d\xi^b = \left[\frac{\rho^2+L^2}{R^2}\right]^{\frac{7-p}{4}} dx^2_{1,p-2} + \left[\frac{R^2}{\rho^2+L^2}\right]^{\frac{7-p}{4}} \Big[d\rho^2+\rho^2\,d\Omega_1^2\Big]\;. \tag{C.2}$$

Let us now study the quadratic fluctuations around the static configuration. As in previous sections, let us denote by χ the scalars transverse to both types of branes. Moreover, we will assume that the Dp-brane probe intersects the Dp-branes of the background along $x^0 \cdots x^{p-2}$ and we will denote the remaining Minkowski coordinates as:

$$\lambda_1 = x^{p-1}\;, \qquad \lambda_2 = x^p\;. \tag{C.3}$$

The lagrangian for the quadratic fluctuations can be obtained straightforwardly by expanding (C.1). Indeed, the contribution from the Born-Infeld lagrangian is:

$$\mathcal{L}_{BI} = -\rho\left[\frac{1}{2}\left[\frac{R^2}{\rho^2+L^2}\right]^{\frac{7-p}{4}}\mathcal{G}^{ab}\,\partial_a\chi\partial_b\chi + \frac{1}{2}\left[\frac{\rho^2+L^2}{R^2}\right]^{\frac{7-p}{4}}\mathcal{G}^{ab}\partial_a\lambda_i\partial_b\lambda_i + \frac{1}{4}F_{ab}F^{ab}\right]\;, \tag{C.4}$$

while the Wess-Zumino term becomes:

$$\mathcal{L}_{WZ} = \left[\frac{\rho^2+L^2}{R^2}\right]^{\frac{7-p}{2}} \epsilon^{ij}\partial_\rho\lambda_i\,\partial_\varphi\lambda_j\;. \tag{C.5}$$

The analysis of the χ fluctuations reduces to the general case of section 2 (see eq. (2.33)) with γ_1 and γ_2 as in eq. (2.14), $p_2 = p$ and $d = p-2$. Notice that in this case the equation for the fluctuations depends on l^2. As argued in the analysis of the $(1|D3 \perp D3)$ intersection, for this real field χ we can restrict ourselves to the case $l \ge 0$. By using the asymptotic limits of the equivalent Schrödinger potential (eq. (2.37)) one readily gets that $\chi \sim \varrho^{\pm l}$ both for $\varrho \to \infty$ and $\varrho \to 0$. The S^l modes in this case will be defined as those modes which behave as ϱ^l for $\varrho \approx 0$ and as ϱ^{-l} for $\varrho \to \infty$. The WKB mass formula for these modes can be obtained from (2.65), namely:

$$M_S^{WKB}(n,l) = 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\left(\frac{7-p}{4}\right)}{\Gamma\left(\frac{5-p}{4}\right)}\sqrt{(n+1)\left(n+\frac{7-p}{5-p}\,l\right)}\;. \tag{C.6}$$

Notice that this result coincides with the exact one (4.5) for $p=3$ and $l=1$. Moreover, the quadratic and linear terms in n for M^2 are also reproduced for $p=3$ and arbitrary l. In table 7 we give the numerical results and the WKB estimates for $\bar{M}^2$ for the intersections of the type $(p-2|Dp \perp Dp)$ for $p=2,4$ and $l=1$.

JHEP04(2006)037

$(0\|D2 \perp D2)$ with $l=1$		
n	WKB	Numerical
0	11.46	11.34
1	36.67	36.54
2	75.63	75.50
3	128.34	128.20
4	194.80	194.66
5	275.01	274.88

$(2\|D4 \perp D4)$ with $l=1$		
n	WKB	Numerical
0	4.31	4.68
1	11.48	11.88
2	21.53	21.94
3	34.45	34.86
4	50.24	50.66
5	68.91	69.34

Table 7: Numerical and WKB values for the S^l modes of the D2-D2 and D4-D4 intersections for $l=1$.

C.1 Scalar fluctuations

The equations of motion of the λ_i's derived from $\mathcal{L}_{BI} + \mathcal{L}_{WZ}$ are:

$$R^{\frac{7-p}{2}}\;\partial_a\Big[\rho\,(\rho^2+L^2)^{\frac{7-p}{4}}\,\mathcal{G}^{ab}\partial_b\lambda_i\Big] - (7-p)\rho\,(\rho^2+L^2)^{\frac{5-p}{2}}\,\epsilon^{ij}\partial_\varphi\lambda_j = 0\;. \tag{C.7}$$

Let us separate variables and define the complex combination w as in the D3-D3 case (eqs. (4.7) and (4.9)). The decoupled equations become:

$$\partial_\varrho\Big[\varrho\,(\varrho^2+1)^{\frac{7-p}{2}}\,\partial_\varrho\,w\Big] + \Bigg[\bar{M}^2\varrho - l^2\,\frac{(\varrho^2+1)^{\frac{7-p}{2}}}{\varrho} + (7-p)l\,\varrho(\varrho^2+1)^{\frac{5-p}{2}}\Bigg]w = 0\;. \tag{C.8}$$

In order to transform this equation into the Schrödinger equation (2.34), let us change variables as:

$$e^y = \varrho\;,\qquad \psi = \Big[1+\varrho^2\Big]^{\frac{7-p}{4}}\;w\;. \tag{C.9}$$

The potential in this case becomes:

$$\begin{aligned} V(y) \,=\, & l^2 + (7-p)(1-l)\frac{e^{2y}}{1+e^{2y}} + \\ & + \frac{1}{4}\,(7-p)(3-p)\,\frac{e^{4y}}{(1+e^{2y})^2} - \bar{M}^2\frac{e^{2y}}{(1+e^{2y})^{\frac{7-p}{2}}}\;. \end{aligned} \tag{C.10}$$

The asymptotic values of V can be readily computed, with the result:

$$\lim_{y\to-\infty} V(y) = l^2\;,\qquad \lim_{y\to+\infty} V(y) = \left(l - \frac{7-p}{2}\right)^2\;. \tag{C.11}$$

From these values and the relation between ψ and w we obtain that for $\varrho \approx 0$:

$$w \approx c_1\varrho^{l} + c_2\varrho^{-l}\;,\qquad (\varrho\approx 0)\;, \tag{C.12}$$

whereas for large ρ we have:

$$w \approx d_1\varrho^{-l} + d_2\varrho^{l+p-7}\;,\qquad (\varrho\to\infty)\;. \tag{C.13}$$

Following the steps of the analysis of the scalar fluctuations in the D3-D3 system, the different modes are characterized by their behaviour at $\varrho \to \infty$, as follows:

$$\begin{aligned} W_+^l \qquad &\Longrightarrow \qquad w \sim \varrho^{-l} \ , \\ W_-^l \qquad &\Longrightarrow \qquad w \sim \varrho^{l+p-7} \ . \end{aligned} \tag{C.14}$$

In addition the modes should not blow up at $\varrho = 0$, i.e. they should behave as $\varrho^{|l|}$ near $\varrho = 0$. Interestingly, the WKB approximation selects the W_+^l or W_-^l modes, depending on the value of l. Indeed, for large ϱ the functions w and ψ are related as $w \sim \varrho^{\frac{p-7}{2}} \psi$ and, within the WKB approach, the wave function ψ vanishes as $\psi \sim \varrho^{-|l-\frac{7-p}{2}|}$ when $\varrho \to \infty$. It follows that w behaves as in the W_+^l branch for $l \geq \frac{7-p}{2}$ and as in the W_-^l modes for $l \leq \frac{7-p}{2}$. To compute the actual values of the WKB energy levels we need to evaluate the coefficients α_i and β_i defined in eqs. (2.59) and (2.60). In the present case these coefficients are:

$$\alpha_1 = 2 \ , \qquad \alpha_2 = 2|\,l\,| \ , \qquad \beta_1 = 5-p \ , \qquad \beta_2 = |\,2l-7+p\,| \ . \tag{C.15}$$

The appearance of the absolute value on the expressions of α_2 and β_2 in (C.15) forces us to consider different ranges of l. For $l \geq \frac{7-p}{2}$ the WKB method selects the W_+^l branch and the corresponding energy levels are:

$$M_{W_+}^{WKB}(n,l) = 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\Big(\frac{7-p}{4}\Big)}{\Gamma\Big(\frac{5-p}{4}\Big)}\,\sqrt{(n+1)\Big(n+\frac{7-p}{5-p}\,(l-1)\Big)} \ , \qquad \Big(l \geq \frac{7-p}{2}\Big) \ . \tag{C.16}$$

This mass spectrum is related to the one corresponding to the transverse scalar excitations as follows:

$$M_{W_+}^{WKB}(n,l) = M_S^{WKB}(n,l-1) \ , \qquad \Big(l \geq \frac{7-p}{2}\Big) \ . \tag{C.17}$$

When $l \leq 0$ the W_- branch is selected by the WKB approach and one gets the following mass formula:

$$M_{W_-}^{WKB}(n,l) = 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\Big(\frac{7-p}{4}\Big)}{\Gamma\Big(\frac{5-p}{4}\Big)}\,\sqrt{(n+1)\Big(n+\frac{7-p}{5-p}\,(1-l)\Big)} \ , \qquad (l \leq 0) \ , \tag{C.18}$$

which is related to the spectrum of the S^l modes as:

$$M_{W_-}^{WKB}(n,l) = M_S^{WKB}(n,1-l) \ , \qquad (l \leq 0) \ . \tag{C.19}$$

We have checked that the relations (C.17) and (C.19) are well satisfied by the masses computed by solving numerically the corresponding fluctuation equations.

For the range $0 \leq l \leq \frac{7-p}{2}$ the WKB method picks up the W_- branch and one gets:

$$M_{W_-}^{WKB}(n,l) = 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\Big(\frac{7-p}{4}\Big)}{\Gamma\Big(\frac{5-p}{4}\Big)}\sqrt{(n+1)\Big(n+2l+\frac{7-p}{5-p}(1-l)\Big)}, \quad \Big(0 \leq l \leq \frac{7-p}{2}\Big). \tag{C.20}$$

We have checked that (C.20) represents fairly well the results obtained numerically.

JHEP04(2006)037

Notice that the expression (C.16) of $M_{W_+}^{WKB}$ vanishes for $n = 0$, $l = 1$. Actually, when the mass M vanishes, the equation (C.8) for the fluctuations can be written as:

$$\varrho^2(1+\varrho^2)\,\partial_\varrho^2 w + [1+(8-p)\varrho^2\,]\,\varrho\,\partial_\varrho w \;-\; l[l-(7-p-l)\varrho^2\,]\,w = 0\;. \tag{C.21}$$

This equation can be mapped into the hypergeometric equation. Actually, one of its solutions is just $w = \rho^{-l}$. The second solution can be written in terms of the hypergeometric function as follows:

$$w = \varrho^l\, F\left(l, \frac{7-p}{2}; 1+l; -\varrho^2\right). \tag{C.22}$$

When $p = 3$ and $l = 1$ this solution reduces to the one written in (4.16), which represents the holomorphic map of the Higgs branch of the D3-D3 intersection.

C.2 Vector fluctuations

Let us analyze the configurations of the gauge field which are given by the same ansatzs as in the D3-D3 intersection. First of all, we consider a configuration as the one displayed in eq. (4.23), in which the only non-vanishing components of the gauge field are A_μ, which depends on a function $\phi(\rho)$. After a short calculation one can verify that the equations of motion of the gauge field reduce to the following equation for ϕ:

$$\partial_\varrho\left(\varrho\,\partial_\varrho\,\phi\right) + \left[\bar{M}^2\,\frac{\varrho}{(1+\varrho^2)^{\frac{7-p}{2}}} - \frac{l^2}{\varrho}\right]\phi = 0\;, \tag{C.23}$$

where we have already used the reduced variables ϱ and $\bar{M}$ defined in eq. (B.20). Eq. (C.23) is just the same as the one satisfied by the transverse scalars and, therefore, the corresponding spectra are identical. Let us next consider an ansatz as in eq. (4.25), which depends on two functions ϕ and $\tilde{\phi}$. The equation for A_ρ in this case becomes:

$$il\partial_\rho\tilde{\phi} - l^2\phi + M^2R^{7-p}\frac{\rho^2}{(\rho^2+L^2)^{\frac{7-p}{2}}}\,\phi = 0\;. \tag{C.24}$$

Moreover, the equation for A_φ is:

$$\partial_\rho\left[\frac{(\rho^2+L^2)^{\frac{7-p}{2}}}{\rho}\left(\partial_\rho\tilde{\phi} + il\phi\right)\right] + \frac{M^2R^{7-p}}{\rho}\,\tilde{\phi} = 0\;. \tag{C.25}$$

The equation for A_μ gives a relation between ϕ and $\tilde{\phi}$, which is the same as in the D3-D3 intersection, namely eq. (4.28). By using this relation the remaining equations reduce to the following equation for ϕ:

$$\partial_\rho\Big[\rho\,\partial_\rho(\rho\phi)\Big] + \left[\frac{\rho^2}{(\rho^2+L^2)^{\frac{7-p}{2}}}\,M^2 - l^2\right]\phi = 0\;. \tag{C.26}$$

This equation becomes again just the same as the one corresponding to the transverse scalars if we redefine the fluctuation as $\hat{\phi} = \rho\phi$. Then, the spectrum of these modes coincides again with that of the transverse scalars. The conclusion we arrive at is that, also in this case, the scalar and vector modes are degenerate in mass, i.e.:

$$M_V(n,l) = M_S(n,l)\;. \tag{C.27}$$

D. Fluctuations of the Dp-D(p+4) system

Let us consider a D(p+4)-brane probe embedded in the background created by a Dp-brane in such a way that the probe fills the (p+1)-dimensional worldvolume of the background brane for $1 \leq p < 5$. If L denotes the distance between both types of branes, the induced metric on the D(p+4) worldvolume is:

$$\mathcal{G}_{ab}d\xi^a d\xi^b = \left[\frac{\rho^2+L^2}{R^2}\right]^{\frac{7-p}{4}} dx^2_{1,p} + \left[\frac{R^2}{\rho^2+L^2}\right]^{\frac{7-p}{4}} \left[d\rho^2 + \rho^2\, d\Omega_3^2\right]. \tag{D.1}$$

The dynamics of the probe is governed by a lagrangian density which is the sum of the Born-Infeld term $\mathcal{L}_{BI}$ and the Wess-Zumino term $\mathcal{L}_{WZ}$. The expression of $\mathcal{L}_{BI}$ is just the same as the one appearing in (B.1) and (C.1). By expanding it around the static configuration, one gets the following expression:

$$\mathcal{L}_{BI} = -\rho^3\sqrt{\tilde{g}}\left[\frac{1}{2}\left[\frac{R^2}{\rho^2+L^2}\right]^{\frac{7-p}{4}} \mathcal{G}^{ab}\,\partial_a\chi\partial_b\chi + \frac{1}{4}F_{ab}F^{ab}\right], \tag{D.2}$$

where χ are the fluctuations of the scalars transverse to the probe and F_{ab} is the strength of the wordlvolume gauge field. The Wess-Zumino lagrangian in this case is:

$$\mathcal{L}_{WZ} = \frac{1}{2}\,P[C^{(p+1)}]\wedge F\wedge F\,. \tag{D.3}$$

Using the expression of the Ramond-Ramond potential $C^{(p+1)}$ given in eq. (2.15) and dropping terms that do not contribute to the equations of motion we get:

$$\mathcal{L}_{WZ} = \left[\frac{\rho^2+L^2}{R^2}\right]^{\frac{7-p}{2}} \epsilon_{ijk}\,\partial_\rho A_i\,\partial_j A_k\,. \tag{D.4}$$

D.1 Scalar fluctuations

The equations of motion for the scalar fields χ are a particular case of those studied in section 2. Indeed, in this case one must take γ_1 and γ_2 as given in eq. (2.14), $p_2 = p+4$ and $d = p$. Using these values in the WKB estimate (2.65), we arrive at the expression:

$$M_S^{WKB}(n,l) = 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\left(\frac{7-p}{4}\right)}{\Gamma\left(\frac{5-p}{4}\right)}\,\sqrt{(n+1)\left(n+\frac{7-p}{5-p}(l+1)\right)}\,. \tag{D.5}$$

In table 8 we compare the numerical values of the masses for the scalar fluctuations with the corresponding WKB estimates.

D.2 Vector fluctuations

The equation of motion of the gauge field derived from $\mathcal{L}_{BI}+\mathcal{L}_{WZ}$ of eqs. (D.2) and (D.4) takes the form:

$$\partial_a\left[\rho^3\sqrt{\tilde{g}}\,F^{ab}\right] - (7-p)\,\frac{\rho}{R^{7-p}}\left[\rho^2+L^2\right]^{\frac{5-p}{2}} \epsilon_{bjk}\,\partial_j A_k = 0\,, \tag{D.6}$$

JHEP04(2006)037

$(1\|D1 \perp D5)$ with $l=0$		
n	WKB	Numerical
0	14.80	14.70
1	49.35	49.22
2	103.63	103.50
3	177.65	177.54
4	271.41	271.30
5	384.91	384.80

$(2\|D2 \perp D6)$ with $l=0$		
n	WKB	Numerical
0	11.46	11.34
1	36.67	36.54
2	75.63	75.50
3	128.34	128.20
4	194.80	194.66
5	275.01	274.88

$(4\|D4 \perp D8)$ with $l=0$		
n	WKB	Numerical
0	4.31	4.68
1	11.48	11.88
2	21.53	21.94
3	34.45	34.86
4	50.24	50.66
5	68.91	69.34

Table 8: Values of $\bar{M}^2$ obtained numerically and with the WKB method for the scalar modes of the D1-D5, D2-D6 and D4-D8 intersections for $l=0$.

where ϵ_{bjk} is non-vanishing only when b is an index along the S^3.

Let us study the solutions of this equation. Following [7], we can expand A_μ and A_ρ in (scalar) spherical harmonics on the S^3, and A_i in vector spherical harmonics. We can construct three classes of vector spherical harmonics on the S^3. One is simply given by $Y_i^l \equiv \nabla_i Y^l$, and the other two, denoted by $Y_i^{l,\pm}$ for $l \geq 1$, transform in the $(\frac{l\mp1}{2}, \frac{l\pm1}{2})$ representation of SO(4) and satisfy:

$$\begin{aligned} \nabla^i \nabla_i Y_j^{l,\pm} - R_j^k Y_k^{l,\pm} &= -(l+1)^2 Y_j^{l,\pm} , \\ \epsilon_{ijk} \nabla_j Y_k^{l,\pm} &= \pm(l+1) Y_i^{l,\pm} , \\ \nabla^i Y_i^{l,\pm} &= 0 , \end{aligned} \tag{D.7}$$

where $R_j^i = 2\delta_j^i$ is the Ricci tensor of a three-sphere of unit radius.

D.2.1 Type I modes

As argued in [7], the modes containing $Y_i^{l,\pm}$ do not mix with the others due to the fact that they are in different representations of SO(4). Accordingly, let us take the ansatz:

$$A_\mu = 0 \,, \qquad A_\rho = 0 \,, \qquad A_i = \Lambda^\pm(x,\rho)\, Y_i^{l,\pm}(S^3) \,. \tag{D.8}$$

The equation of motion (D.6) reduces to the following equation for $\Lambda^\pm(x,\rho)$:

$$\begin{aligned} \rho R^{7-p} \partial_\mu \partial^\mu \Lambda^\pm + \partial_\rho \Big[\rho (\rho^2+L^2)^{\frac{7-p}{2}} \partial_\rho \Lambda^\pm \Big] - (l+1)^2 \frac{(\rho^2+L^2)^{\frac{7-p}{2}}}{\rho} \Lambda^\pm \mp \\ \mp (7-p)(l+1)\, \rho\, (\rho^2+L^2)^{\frac{5-p}{2}} \Lambda^\pm = 0 \,. \end{aligned} \tag{D.9}$$

Let us separate variables in (D.9) as in previous cases, namely:

$$\Lambda^\pm(x,\rho) = e^{ikx}\, \xi^\pm(\rho) \,. \tag{D.10}$$

Moreover, we shall introduce the reduced quantities ϱ and $\bar{M}$, defined as in (B.20). By changing variables as:

$$e^y = \varrho \,, \qquad \psi^\pm = \Big[1+\varrho^2\Big]^{\frac{7-p}{4}} \xi^\pm \,, \tag{D.11}$$

JHEP04(2006)037

we can convert the fluctuation equation (D.9) into a Schrödinger equation for $\psi^\pm$, with the potential $V_\pm$ given by:

$$V_\pm(y) = \Big(l+1\Big)^2 \pm (7-p)\Big(l+1\pm 1\Big)\frac{e^{2y}}{1+e^{2y}} + \\ +\frac{1}{4}(7-p)(3-p)\frac{e^{4y}}{(1+e^{2y})^2} - \bar{M}^2\frac{e^{2y}}{(1+e^{2y})^{\frac{7-p}{2}}} \,. \tag{D.12}$$

By looking at the asymptotic value of the potential $V_\pm$ at $y \to \pm\infty$ we can get the behaviour of the fluctuations $\xi^\pm$ at $\varrho \approx 0, \infty$. Indeed, from the above potentials we obtain:

$$\lim_{y\to-\infty} V_\pm(y) = \Big(l+1\Big)^2 \,, \qquad \lim_{y\to+\infty} V_\pm(y) = \Big(l+1\pm\frac{7-p}{2}\Big)^2 \,. \tag{D.13}$$

From these values one can prove that for $\varrho \approx 0$:

$$\xi^\pm \approx c_1\varrho^{l+1} + c_2\varrho^{-(l+1)} \,, \qquad (\varrho \approx 0) \,, \tag{D.14}$$

whereas for $\varrho \to \infty$ one gets:

$$\begin{aligned} \xi^+ &\approx d_1^+\varrho^{-(l+8-p)} + d_2^+\varrho^{l+1} \,, \\ \xi^- &\approx d_1^-\varrho^{-(l+1)} + d_2^-\varrho^{l+p-6} \,, \qquad (\varrho\to\infty) \,. \end{aligned} \tag{D.15}$$

Obviously, the regular solutions should behave as ϱ^{l+1} as $\varrho \to 0$. The regularity at the UV requires also the vanishing of $\xi^\pm$ when $\varrho \to \infty$. For the ξ^+ fluctuation this requirement is only satisfied when $d_2^+ = 0$ in (D.15). This condition defines the so-called I_+^l modes. On the other hand, for the ξ^- fluctuation we have clearly two possibilities. The modes with $d_2^- = 0$ for $l \geq 1$ will be denoted by I_-^l, whereas those with $d_1^- = 0$ for $1 \leq l < 6-p$ will be called $\tilde{I}_-^l$. Summarizing, the different behaviours at $\varrho \to \infty$ are:

$$\begin{aligned} I_+^l &\quad\Longrightarrow\quad \xi^+ \sim \varrho^{-(l+8-p)} \,, \qquad (l\geq 1) \,, \\ I_-^l &\quad\Longrightarrow\quad \xi^- \sim \varrho^{-(l+1)} \,, \qquad (l\geq 1) \,, \\ \tilde{I}_-^l &\quad\Longrightarrow\quad \xi^- \sim \varrho^{l+p-6} \,, \qquad (1\leq l < 6-p) \,. \end{aligned} \tag{D.16}$$

In order to get information about the mass levels for these modes, let us compute the different spectra in the WKB approximation. One can readily verify that the WKB method selects always the I_+^l modes of the ξ^+ fluctuation, whereas it picks up one of the two branches of the ξ^- modes, depending on the value of l. Indeed, if $l \geq \frac{5-p}{2}$, the I_-^l branch is selected, while the $\tilde{I}_-^l$ modes are picked up otherwise. The corresponding WKB mass levels are given by:

$$\begin{aligned} M_{I_\pm}^{WKB}(n,l) &= 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\big(\frac{7-p}{4}\big)}{\Gamma\big(\frac{5-p}{4}\big)}\,\sqrt{(n+1)\,\Big(n+\frac{7-p}{5-p}\,(l+1\pm 1)\Big)} \,, \\ M_{\tilde{I}_-}^{WKB}(n,l) &= 2\sqrt{\pi}\,\frac{L^{\frac{5-p}{2}}}{R^{\frac{7-p}{2}}}\,\frac{\Gamma\big(\frac{7-p}{4}\big)}{\Gamma\big(\frac{5-p}{4}\big)}\,\sqrt{(n+1)\,\Big(n+\frac{7-p}{5-p}+\frac{3-p}{5-p}\,(l+1)\Big)} \,, \end{aligned} \tag{D.17}$$

JHEP04(2006)037

where we have assumed that for each case l varies in the range just discussed. By comparing eqs. (D.17) and (D.5) we conclude that:

$$M_{I_\pm}(n,l) = M_S(n, l \pm 1) \ . \tag{D.18}$$

Actually, the relation (D.18) is satisfied by the masses found numerically with large accuracy and, therefore it seems to hold beyond the WKB approximation[3]. Moreover, the WKB formula in (D.17) for the masses of the $\tilde{I}^l_-$ modes reproduces reasonably well the values found in the numerical calculations.

D.2.2 Type II modes

As before we shall take the ansatz:

$$A_\mu = \phi_\mu(x,\rho)\, Y^l(S^3)\,, \qquad A_\rho = 0\,, \qquad A_i = 0\,, \tag{D.19}$$

where $\partial^\mu \phi_\mu = 0$. The equations of motion for A_ρ and A_i are automatically satisfied. Let us, expanding as before in a plane wave basis, represent ϕ_μ as:

$$\phi_\mu = \xi_\mu\, e^{ikx}\, \chi(\rho)\,, \tag{D.20}$$

with ξ_μ being a constant vector satisfying $k^\mu \xi_\mu = 0$. The equation for A_μ yields:

$$\partial_\varrho(\varrho^3 \partial_\varrho \chi) + \bar{M}^2 \frac{\varrho^3}{(1+\varrho^2)^{\frac{7-p}{2}}}\, \chi - l\,(l+2)\,\chi = 0\,, \tag{D.21}$$

which is the same equation as for the transverse scalar modes. Therefore, we conclude that

$$M_{II}(n,l) = M_S(n,l) \ . \tag{D.22}$$

D.2.3 Type III modes

Let us take the following form for the gauge field:

$$A_\mu = 0 \ , \qquad A_\rho = \phi(x,\rho)\, Y^l(S^3) \ , \qquad A_i = \tilde{\phi}(x,\rho)\, Y^l_i(S^3) \ . \tag{D.23}$$

The equation for A_ρ becomes:

$$\rho^3\, R^{7-p}\, \partial_\mu \partial^\mu \phi \,+\, l(l+2)\,\rho\,(\rho^2 + L^2)^{\frac{7-p}{2}}\, (\partial_\rho \tilde{\phi} - \phi) = 0\,. \tag{D.24}$$

The equation for A_μ can be written as:

$$\partial_\mu \Big[l(l+2)\,\rho\,\tilde{\phi} - \partial_\rho(\rho^3 \phi) \Big] = 0\,. \tag{D.25}$$

Expanding ϕ and $\tilde{\phi}$ in a plane wave basis one can write:

$$l(l+2)\tilde{\phi} = \frac{1}{\rho} \partial_\rho(\rho^3 \phi) \ . \tag{D.26}$$

[3]For $p = 3$ the relation (D.18) is exactly satisfied by the analytical spectra found in [7].

JHEP04(2006)037

For $l \neq 0$, one can use this relation to eliminate $\tilde{\phi}$ in favor of ϕ. The equation of motion of A_ρ becomes:

$$\partial_\rho \left(\frac{1}{\rho}\, \partial_\rho(\rho^3\, \phi) \right) - l(l+2)\, \phi + R^{7-p} \frac{\rho^2}{(\rho^2 + L^2)^{\frac{7-p}{2}}}\, \partial_\mu\, \partial^\mu \phi = 0\,. \tag{D.27}$$

The equation for A_i results equivalent to the above one.

Let us separate variables as in the Dp-D(p+2) case, namely:

$$\phi(x,\rho) = e^{ikx}\, \zeta(\rho)\,. \tag{D.28}$$

In terms of the reduced quantities ϱ and $\bar{M}$ introduced before in (B.20), and by changing variables as:

$$e^y = \varrho\ , \qquad \psi = \varrho^2\, \zeta\,, \tag{D.29}$$

we can convert the fluctuation equation into a Schrödinger equation, where the potential $V(y)$ is given by:

$$V(y) = (l+1)^2 - \bar{M}^2 \frac{e^{2y}}{(1+e^{2y})^{\frac{7-p}{2}}}\,. \tag{D.30}$$

This potential is just the same as the one corresponding to the scalar fluctuations. It follows that the masses of these fluctuations are the same as those corresponding to the scalar modes, i.e.:

$$M_{III}(n,l) = M_S(n,l)\ . \tag{D.31}$$

E. Fluctuations of the F1-Dp systems

Let us now consider the intersection $(0|F1 \perp Dp)$. We will treat the fundamental string as background and the Dp-brane as a probe. The corresponding near-horizon supergravity solution is:

$$\begin{aligned} ds^2 &= \frac{r^6}{R^6}\, (\,-\,dt^2 + (dx^1)^2\,) + d\vec{y}\cdot d\vec{y}\ , \\ e^{-\phi} &= \frac{R^3}{r^3}\ , \end{aligned} \tag{E.1}$$

where R is given in (2.18), $\vec{y} = (y^1, \ldots, y^8)$ and $r^2 = \vec{y}^{\,2}$. We shall place now a Dp-brane at a constant value of x^1 and take the following set of worldvolume coordinates:

$$\xi^a = (t, y^1, \ldots, y^p)\ . \tag{E.2}$$

As before, we shall denote by $\vec{z}$ the vector formed by the $8-p$ coordinates $(y^{p+1}, \ldots, y^8)$. If $\rho^2 = (y^1)^2 + \cdots + (y^p)^2$ and if the Dp-brane is located at $|\vec{z}| = L$, the induced metric is:

$$ds_I^2 = -\left[\frac{\rho^2 + L^2}{R^2}\right]^3 dt^2 + d\rho^2 + \rho^2\, d\Omega_{p-1}^2\ , \tag{E.3}$$

where we have assumed that $p > 1$. We shall limit ourselves here to study the fluctuations of the scalars χ transverse to both the F1 and the Dp-brane. These fluctuations are governed

JHEP04(2006)037

$(0\|F1 \perp D2)$ with $l=1$		
n	WKB	Numerical
0	14.80	14.70
1	49.35	49.22
2	103.63	103.50
3	177.65	177.54
4	271.41	271.30
5	384.91	384.80

$(0\|F1 \perp D5)$ with $l=0$		
n	WKB	Numerical
0	22.21	27.06
1	64.15	69.40
2	125.84	131.36
3	207.26	213.02
4	308.43	314.36
5	429.33	435.40

Table 9: $\bar{M}^2$ for the transverse scalar fluctuations of F1-D2 ($l=1$) and F1-D5 ($l=0$) obtained numerically and with the WKB approximation.

by the lagrangian (2.27) for $p_2 = p$, $d=0$ and for the γ_i exponents written in eq. (2.18). By changing variables as $e^y = \varrho$ and $\psi = \varrho^{\frac{p-2}{2}}\xi$, we can convert the fluctuation into the Schrödinger equation (2.34) with potential:

$$V(y) = \Big(l + \frac{p-2}{2}\Big)^2 - \bar{M}^2\,\frac{e^{2y}}{(e^{2y}+1)^3}\ . \tag{E.4}$$

Notice that the potential (E.4) is invariant if we simultaneously change $l \to l+1$ and $p \to p-2$. This means that the spectrum of the $(0|F1 \perp Dp)$ intersection at angular quantum number l is equivalent to that of $(0|F1 \perp D(p-2))$ at quantum number $l+1$. For this system the corresponding WKB mass levels are:

$$\bar{M}_{WKB} = \pi\sqrt{(n+1)\Big(n + \frac{3}{4}(p-2) + \frac{3l}{2}\Big)}\,. \tag{E.5}$$

As discussed in section 2.5, one can prove from the asymptotic values of the potential (E.4) that, for both $\varrho \to 0$ and $\varrho \to \infty$, the fluctuation ξ behaves as $\xi \sim \varrho^{\gamma}$ with $\gamma = l, -(l+p-2)$. We will select numerically the regular solutions as those which behave as ϱ^l for small ϱ and as $\varrho^{-(l+p-2)}$ for large ϱ. In table 9 we collect some numerical results and the corresponding WKB estimates for some $(0|F1 \perp Dp)$ intersections. By looking at the potential of the equivalent Schrödinger problems, we notice that the transverse fluctuations of the $(0|F1 \perp D3)$ intersection are equivalent to those of the $(0|D1 \perp D3)$ configuration, while the $(0|F1 \perp D4)$ intersection is equivalent to $(0|D1 \perp D5)$.

F. Fluctuations of the M-theory intersections

According to the analysis performed in section 2 the basic supersymmetric orthogonal intersections of M-theory are $(1|M2 \perp M5)$, $(3|M5 \perp M5)$ and $(0|M2 \perp M2)$. The M2 and M5 eleven-dimensional near-horizon metrics are:

$$\begin{aligned} ds^2_{M2} &= \frac{r^4}{R^4}\,(-dt^2 + (dx^1)^2 + (dx^2)^2) + \frac{R^2}{r^2}\,d\vec{y}\cdot d\vec{y}\ ,\\ ds^2_{M5} &= \frac{r}{R}\,(-dt^2 + (dx^1)^2 + \cdots + (dx^5)^2) + \frac{R^2}{r^2}\,d\vec{y}\cdot d\vec{y}\ , \end{aligned} \tag{F.1}$$

JHEP04(2006)037

where the radii R are given in eqs. (2.20) and (2.23). In this appendix we will restrict ourselves to study the fluctuations of the transverse scalars for the three BPS intersections listed in (2.22) and (2.25). For these particular modes, the worldvolume action can be taken to be the square root of the induced metric, as it was done in section 2. We will verify that the corresponding differential equations for these M-theory systems are identical to some of the ones already studied for the type II theory, as expected naturally from the relation between these two theories.

F.1 $(1|M2 \perp M5)$ intersection

Let us consider a M5-brane probe in the M2-brane background written above and let us take the worldvolume coordinates as $\xi^a = (t, x^1, y^1, y^2, y^3, y^4)$. We shall assemble the orthogonal coordinates in the vector $\vec{z} = (y^5, y^6, y^7, y^8)$. For an embedding with x^2 constant and $|\vec{z}| = L$, we have the following induced metric:

$$ds_I^2 = \left[\frac{\rho^2 + L^2}{R^2}\right]^2 (-dt^2 + (dx^1)^2) + \frac{R^2}{\rho^2 + L^2}(d\rho^2 + \rho^2 d\Omega_3^2) , \tag{F.2}$$

where we have used spherical coordinates to parametrize the (y^1, y^2, y^3, y^4) variables. Notice that for $L = 0$, the above metric corresponds to an $AdS_2 \times S^3$ defect of the $AdS_4 \times S^7$ geometry. The equation for the fluctuations around this configuration is:

$$\frac{R^6}{(\rho^2 + L^2)^3}\, \partial^\mu \partial_\mu \chi + \frac{1}{\rho^3}\partial_\rho (\rho^3 \partial_\rho \chi) + \frac{1}{\rho^2}\nabla^i \nabla_i \chi = 0 , \tag{F.3}$$

which is just the same equation as that of the transverse scalars in the D1-D5 system. Therefore, the corresponding mass levels for these modes are exactly the same as in the D1-D5 intersection. Notice that, in order to study the full set of fluctuations, one should analyze the complete PST action [26], something that we will not attempt to do here.

F.2 $(3|M5 \perp M5)$ intersection

We now consider a M5-brane probe in the M5-brane geometry. The worldvolume coordinates are $(t, x^1, x^2, x^3, y^1, y^2)$ and the orthogonal space is parametrized by the vector $\vec{z} = (y^3, y^4, y^5)$. For $|\vec{z}| = L$ and constant x^4 and x^5, the induced metric is:

$$ds_I^2 = \left[\frac{\rho^2 + L^2}{R^2}\right]^{\frac{1}{2}} (-dt^2 + (dx^1)^2 + \cdots + (dx^3)^2) + \frac{R^2}{\rho^2 + L^2}(d\rho^2 + \rho^2 d\Omega_1^2) . \tag{F.4}$$

For $L = 0$ this metric corresponds to an $AdS_5 \times S^1$ defect on the $AdS_7 \times S^4$ background geometry. The equation for the fluctuations is:

$$\frac{R^2}{(\rho^2 + L^2)^{\frac{3}{2}}}\, \partial^\mu \partial_\mu \chi + \frac{1}{\rho}\partial_\rho (\rho\, \partial_\rho \chi) + \frac{1}{\rho^2}\nabla^i \nabla_i \chi = 0 , \tag{F.5}$$

which is identical to the one corresponding to the transverse scalars of the D4-D4 system.

JHEP04(2006)037

F.3 (0|M2 $\perp$ M2) intersection

Let us put a M2-brane probe in the M2 geometry and take (t, y^1, y^2) as worldvolume coordinates. Now $\vec{z} = (y^3, \ldots, y^8)$ and we consider an embedding at x^1 and x^2 constant and $|\vec{z}| = L$. the induced metric is:

$$ds_I^2 = -\left[\frac{\rho^2 + L^2}{R^2}\right]^2 dt^2 + \frac{R^2}{\rho^2 + L^2}\,(d\rho^2 + \rho^2\, d\Omega_1^2)\ , \tag{F.6}$$

which for $L = 0$ is just $AdS_2 \times S^1$. The equation for the fluctuations becomes:

$$\frac{R^6}{(\rho^2 + L^2)^3}\ \partial^0 \partial_0\, \chi + \frac{1}{\rho}\,\partial_\rho\,(\rho\,\partial_\rho \chi) + \frac{1}{\rho^2}\,\nabla^i \nabla_i\, \chi = 0\ , \tag{F.7}$$

and is identical to the one for the transverse scalars of the F1-D2 system.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Adv. Theor. Math. Phys.* **2** (1998) 231 [hep-th/9711200].

[2] For a review see, O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111].

[3] J.M. Maldacena, *Wilson loops in large-N field theories*, *Phys. Rev. Lett.* **80** (1998) 4859 [hep-th/9803002];
S.-J. Rey and J.-T. Yee, *Macroscopic strings as heavy quarks in large-N gauge theory and anti-de Sitter supergravity*, *Eur. Phys. J.* **C 22** (2001) 379 [hep-th/9803001].

[4] A. Karch and L. Randall, *Locally localized gravity*, *JHEP* **05** (2001) 008 [hep-th/0011156];
Open and closed string interpretation of SUSY CFT's on branes with boundaries, *JHEP* **06** (2001) 063 [hep-th/0105132].

[5] A. Karch and E. Katz, *Adding flavor to AdS/CFT*, *JHEP* **06** (2002) 043 [hep-th/0205236];
A. Karch, E. Katz and N. Weiner, *Hadron masses and screening from AdS wilson loops*, *Phys. Rev. Lett.* **90** (2003) 091601 [hep-th/0211107].

[6] A. Fayyazuddin and M. Spalinski, *Large-N superconformal gauge theories and supergravity orientifolds*, *Nucl. Phys.* **B 535** (1998) 219 [hep-th/9805096];
O. Aharony, A. Fayyazuddin and J.M. Maldacena, *The large-N limit of $N = 2, 1$ field theories from three-branes in F-theory*, *JHEP* **07** (1998) 013 [hep-th/9806159];
M. Graña and J. Polchinski, *Gauge/gravity duals with holomorphic dilaton*, *Phys. Rev.* **D 65** (2002) 126005 [hep-th/0106014];
M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda and R. Marotta, *$N = 2$ gauge theories on systems of fractional D3/D7 branes*, *Nucl. Phys.* **B 621** (2002) 157 [hep-th/0107057];
M. Bertolini, P. Di Vecchia, G. Ferretti and R. Marotta, *Fractional branes and $N = 1$ gauge theories*, *Nucl. Phys.* **B 630** (2002) 222 [hep-th/0112187].

[7] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Meson spectroscopy in AdS/CFT with flavour*, *JHEP* **07** (2003) 049 [hep-th/0304032].

[8] T. Sakai and J. Sonnenschein, *Probing flavored mesons of confining gauge theories by supergravity*, *JHEP* **09** (2003) 047 [hep-th/0305049].

JHEP04(2006)037

[9] J. Babington, J. Erdmenger, N.J. Evans, Z. Guralnik and I. Kirsch, *Chiral symmetry breaking and pions in non-supersymmetric gauge/gravity duals*, *Phys. Rev.* **D 69** (2004) 066007 [hep-th/0306018];
R. Apreda, J. Erdmenger and N. Evans, *Scalar effective potential for D7 brane probes which break chiral symmetry*, hep-th/0509219;
R. Apreda, J. Erdmenger, N. Evans, J. Grosse and Z. Guralnik, *Instantons on D7 brane probes and AdS/CFT with flavour*, hep-th/0601130.

[10] M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Towards a holographic dual of large-N_c QCD*, *JHEP* **05** (2004) 041 [hep-th/0311270];
J.L.F. Barbón, C. Hoyos, D. Mateos and R.C. Myers, *The holographic life of the η'*, *JHEP* **10** (2004) 029 [hep-th/0404260];
A. Armoni, *Witten-veneziano from Green-Schwarz*, *JHEP* **06** (2004) 019 [hep-th/0404248];
J.L. Hovdebo, M. Kruczenski, D. Mateos, R.C. Myers and D.J. Winters, *Holographic mesons: adding flavor to the AdS/CFT duality*, *Int. J. Mod. Phys.* **A 20** (2005) 3428.

[11] P. Ouyang, *Holomorphic D7-branes and flavored $N = 1$ gauge theories*, *Nucl. Phys.* **B 699** (2004) 207 [hep-th/0311084];
T.S. Levi and P. Ouyang, *Mesons and flavor on the conifold*, hep-th/0506021.

[12] X.-J. Wang and S. Hu, *Intersecting branes and adding flavors to the Maldacena-Núñez background*, *JHEP* **09** (2003) 017 [hep-th/0307218].

[13] C. Núñez, A. Paredes and A.V. Ramallo, *Flavoring the gravity dual of $N = 1$ Yang-Mills with probes*, *JHEP* **12** (2003) 024 [hep-th/0311201].

[14] S. Hong, S. Yoon and M.J. Strassler, *Quarkonium from the fifth dimension*, *JHEP* **04** (2004) 046 [hep-th/0312071].

[15] N.J. Evans and J.P. Shock, *Chiral dynamics from AdS space*, *Phys. Rev.* **D 70** (2004) 046002 [hep-th/0403279];
N. Evans, J. Shock and T. Waterson, *D7 brane embeddings and chiral symmetry breaking*, *JHEP* **03** (2005) 005 [hep-th/0502091];
J.P. Shock, *Canonical coordinates and meson spectra for scalar deformed $N = 4$ SYM from the AdS/CFT correspondence*, hep-th/0601025.

[16] K. Ghoroku and M. Yahiro, *Chiral symmetry breaking driven by dilaton*, *Phys. Lett.* **B 604** (2004) 235 [hep-th/0408040]; *Holographic model for mesons at finite temperature*, hep-ph/0512289;
K. Ghoroku, T. Sakaguchi, N. Uekusa and M. Yahiro, *Flavor quark at high temperature from a holographic model*, *Phys. Rev.* **D 71** (2005) 106002 [hep-th/0502088];
I. Brevik, K. Ghoroku and A. Nakamura, *Meson mass and confinement force driven by dilaton*, *Int. J. Mod. Phys.* **D 15** (2006) 57 [hep-th/0505057].

[17] D. Arean, D.E. Crooks and A.V. Ramallo, *Supersymmetric probes on the conifold*, *JHEP* **11** (2004) 035 [hep-th/0408210].

[18] S. Kuperstein, *Meson spectroscopy from holomorphic probes on the warped deformed conifold*, *JHEP* **03** (2005) 014 [hep-th/0411097].

[19] T. Sakai and S. Sugimoto, *Low energy hadron physics in holographic QCD*, *Prog. Theor. Phys.* **113** (2005) 843 [hep-th/0412141]; *More on a holographic dual of QCD*, *Prog. Theor. Phys.* **114** (2006) 1083 [hep-th/0507073].

JHEP04(2006)037

[20] D. Arean, A. Paredes and A.V. Ramallo, *Adding flavor to the gravity dual of non-commutative gauge theories*, *JHEP* **08** (2005) 017 [hep-th/0505181].

[21] O. DeWolfe, D.Z. Freedman and H. Ooguri, *Holography and defect conformal field theories*, *Phys. Rev.* **D 66** (2002) 025009 [hep-th/0111135].

[22] J. Erdmenger, Z. Guralnik and I. Kirsch, *Four-dimensional superconformal theories with interacting boundaries or defects*, *Phys. Rev.* **D 66** (2002) 025020 [hep-th/0203020].

[23] K. Skenderis and M. Taylor, *Branes in AdS and pp-wave spacetimes*, *JHEP* **06** (2002) 025 [hep-th/0204054].

[24] N.R. Constable, J. Erdmenger, Z. Guralnik and I. Kirsch, *Intersecting D3-branes and holography*, *Phys. Rev.* **D 68** (2003) 106007 [hep-th/0211222].

[25] I. Kirsch, *Generalizations of the AdS/CFT correspondence*, *Fortschr. Phys.* **52** (2004) 724.

[26] P. Pasti, D.P. Sorokin and M. Tonin, *Covariant action for a $D = 11$ five-brane with the chiral field*, *Phys. Lett.* **B 398** (1997) 41 [hep-th/9701037];
I.A. Bandos, K. Lechner, A. Nurmagambetov, P. Pasti, D. Sorokin and M. Tonin, *Covariant action for the super-five-brane of M-theory*, *Phys. Rev. Lett.* **78** (1997) 4332 [hep-th/9701149];
D.P. Sorokin, *On some features of the M5-brane*, hep-th/9807050.

[27] J.P. Gauntlett, D.A. Kastor and J.H. Traschen, *Overlapping branes in M-theory*, *Nucl. Phys.* **B 478** (1996) 544 [hep-th/9604179].

[28] J. Erdmenger, Z. Guralnik, R. Helling and I. Kirsch, *A world-volume perspective on the recombination of intersecting branes*, *JHEP* **04** (2004) 064 [hep-th/0309043].

[29] J.A. Minahan, *Glueball mass spectra and other issues for supergravity duals of QCD models*, *JHEP* **01** (1999) 020 [hep-th/9811156].

[30] J.G. Russo and K. Sfetsos, *Rotating D3 branes and QCD in three dimensions*, *Adv. Theor. Math. Phys.* **3** (1999) 131 [hep-th/9901056].

[31] C. Csáki, H. Ooguri, Y. Oz and J. Terning, *Glueball mass spectrum from supergravity*, *JHEP* **01** (1999) 017 [hep-th/9806021];
R. de Mello Koch, A. Jevicki, M. Mihailescu and J.P. Nunes, *Evaluation of glueball masses from supergravity*, *Phys. Rev.* **D 58** (1998) 105009 [hep-th/9806125].

[32] N. Itzhaki, J.M. Maldacena, J. Sonnenschein and S. Yankielowicz, *Supergravity and the large-N limit of theories with sixteen supercharges*, *Phys. Rev.* **D 58** (1998) 046004 [hep-th/9802042].

[33] J. Erlich, E. Katz, D.T. Son and M.A. Stephanov, *QCD and a holographic model of hadrons*, *Phys. Rev. Lett.* **95** (2005) 261602 [hep-ph/0501128];
L. Da Rold and A. Pomarol, *Chiral symmetry breaking from five dimensional spaces*, *Nucl. Phys.* **B 721** (2005) 79 [hep-ph/0501218];
G.F. de Teramond and S.J. Brodsky, *The hadronic spectrum of a holographic dual of QCD*, *Phys. Rev. Lett.* **94** (2005) 201601 [hep-th/0501022];
G. Siopsis, *Correspondence principle for a brane in Minkowski space and vector mesons*, *JHEP* **10** (2005) 059 [hep-th/0503245];
N. Mahajan, *Revisiting 5d chiral symmetry breaking and holographic QCD models*, *Phys. Lett.* **B 623** (2005) 119 [hep-ph/0506098];

JHEP04(2006)037

J. Hirn and V. Sanz, *Interpolating between low and high energy QCD via a 5d Yang-Mills model*, *JHEP* **12** (2005) 030 [hep-ph/0507049];
T. Hambye, B. Hassanain, J. March-Russell and M. Schvellinger, *On the $\Delta I = 1/2$ rule in holographic QCD*, hep-ph/0512089.

[34] E. Schreiber, *Excited mesons and quantization of string endpoints*, hep-th/0403226.

[35] Y. Susaki, Y. Takayama and K. Yoshida, *Open semiclassical strings and long defect operators in AdS/dCFT correspondence*, *Phys. Rev.* **D 71** (2005) 126006 [hep-th/0410139];
Integrability and higher loops in AdS/dCFT correspondence, *Phys. Lett.* **B 624** (2005) 115 [hep-th/0504209];
K. Okamura, Y. Takayama and K. Yoshida, *Open spinning strings and AdS/dCFT duality*, *JHEP* **01** (2006) 112 [hep-th/0511139].

[36] P. Lee and J.-W. Park, *Open strings in pp-wave background from defect conformal field theory*, *Phys. Rev.* **D 67** (2003) 026002 [hep-th/0203257].

[37] O. DeWolfe and N. Mann, *Integrable open spin chains in defect conformal field theory*, *JHEP* **04** (2004) 035 [hep-th/0401041].

[38] S.A. Cherkis and A. Hashimoto, *Supergravity solution of intersecting branes and AdS/CFT with flavor*, *JHEP* **11** (2002) 036 [hep-th/0210105];
H. Nastase, *On Dp-Dp+4 systems, QCD dual and phenomenology*, hep-th/0305069;
B.A. Burrington, J.T. Liu, L.A. Pando Zayas and D. Vaman, *Holographic duals of flavored $N = 1$ super Yang-Mills: beyond the probe approximation*, *JHEP* **02** (2005) 022 [hep-th/0406207];
J. Erdmenger and I. Kirsch, *Mesons in gauge/gravity dual with large number of fundamental fields*, *JHEP* **12** (2004) 025 [hep-th/0408113];
I. Kirsch and D. Vaman, *The D3/D7 background and flavor dependence of Regge trajectories*, *Phys. Rev.* **D 72** (2005) 026007 [hep-th/0505164].

[39] R. Casero, C. Núñez and A. Paredes, *Towards the string dual of $N = 1$ sqcd-like theories*, hep-th/0602027.

[40] A. Karch, A. O'Bannon and K. Skenderis, *Holographic renormalization of probe D-branes in AdS/CFT*, hep-th/0512125.

[41] J. Erdmenger, J. Grosse and Z. Guralnik, *Spectral flow on the Higgs branch and AdS/CFT duality*, *JHEP* **06** (2005) 052 [hep-th/0502224];
R. Apreda, J. Erdmenger, N. Evans and Z. Guralnik, *Strong coupling effective Higgs potential and a first order thermal phase transition from AdS/CFT duality*, *Phys. Rev.* **D 71** (2005) 126002 [hep-th/0504151].

[42] S. Yamaguchi, *AdS branes corresponding to superconformal defects*, *JHEP* **06** (2003) 002 [hep-th/0505007].

[43] F. Canoura, J.D. Edelstein, L.A.P. Zayas, A.V. Ramallo and D. Vaman, *Supersymmetric branes on $AdS_5 \times Y^{p,q}$ and their field theory duals*, *JHEP* **03** (2006) 101 [hep-th/0512087].

[44] F. Canoura, A. Paredes and A.V. Ramallo, *Supersymmetric defects in the Maldacena-Núñez background*, *JHEP* **09** (2005) 032 [hep-th/0507155].

JHEP04(2006)037